## Chapter 4

## Principles of Steady - State Heat Transfer

Heat transfer is occurring in many chemical and separation processes as a consequence of a temperature difference. In Chapter 4, the following problem modules explain the heat transfer processes involved in fuel cell vehicles and in the processes for producing fuel for fuel cells.

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### Example 4.1-1: Heat Loss through a Stainless Steel Bipolar Plate

Calculate the heat flux through a stainless steel bipolar plate in a polymer-electrolyte membrane fuel cell with a thickness of 4.5 mm. The fuel cell is operating at a temperature of 80 ºC during the summer season in Houghton, Michigan where the temperature is 70 ºF.

### Strategy

The equation for the heat flux obtained from Fourier's Law can be used to obtain the solution to this problem.

### Solution

Equation 4.1-10 of Geankoplis is defining the heat transfer per unit area as follows:

$$\frac{q}{A} = \frac{k}{x_2 - x_1}\left(T_1 - T_2\right)$$

We can substitute the values given in the problem statement into this equation, but first we need to convert the temperature outside the fuel cell to ºC:

$$T\left(^\circ C\right) = \frac{T\left(^\circ F\right) - 32}{1.8} = \frac{70^\circ F - 32}{1.8}$$

$$T = \underline{\hspace{3cm}}\ ^\circ C$$

Entering the temperatures inside and outside the fuel cell stack into the heat transfer equation, as well as the thickness of the bipolar plate represented by $x_2 - x_1$, we get:

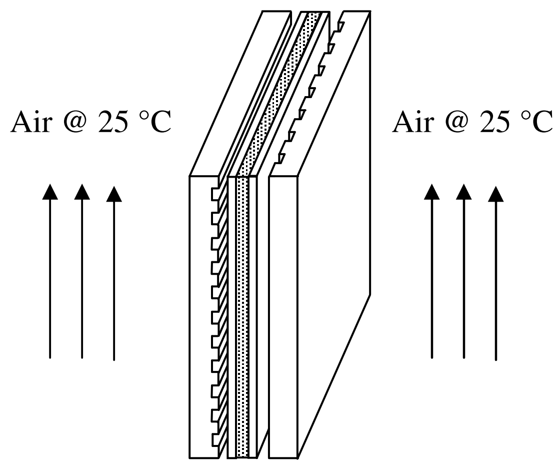
$$\frac{q}{A} = \frac{\underline{\hspace{3cm}}\ \frac{W}{m \cdot K}}{\underline{\hspace{3.5cm}}\ m}\left(\underline{\hspace{3cm}}\ K - 294.25\ K\right)$$

$$\boxed{\frac{q}{A} = \underline{\hspace{5cm}}\ \frac{W}{m^2}}$$

The thermal conductivity of steel was obtained from Table 4.1-1 of Geankoplis.

### Example 4.3-1: Cooling of a Fuel Cell

Air at a temperature of 25 °C is being used for cooling a single cell fuel cell. The convective heat transfer coefficient of the air is $61.2 \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ and is capable of removing heat at a rate of 183.6 W. What would be the dimensions of the square surface of the fuel cell if its temperature must not exceed 50°C?




### Strategy

The heat transfer rate by convection can be obtained using Newton's Law of Cooling.

### Solution

The heat flux $\dot{q}$ when heat is being transferred by forced convection is defined as follows:

$$\dot{q} = hA\left(T_S - T_\infty\right)$$

where:

$\dot{q} =$ heat transfer rate, $\frac{\text{W}}{\text{m}^2}$

$h =$ convective heat transfer coefficient, $\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$

$T_S =$ temperature on the surface of the object, °C

$T_\infty =$ temperature of the air, °C

A = surface area of the fuel cell, $\text{m}^2$

To determine the dimensions of the surface of the fuel cell, we can substitute the given temperatures and heat transfer rate and solve for the area A to yield:

$$A = \frac{\underline{\hspace{2em}}}{\underline{\hspace{8em}}} = \frac{\underline{\hspace{4em}}\ \text{W}}{\underline{\hspace{4em}}\ \frac{\text{W}}{\text{m}^2 \cdot {}^\circ\text{C}}\left(50^\circ\text{C} - 25^\circ\text{C}\right)}$$

$$A = 0.12\ \text{m}^2$$

Since the heat is being removed from the fuel cell through both the left and right faces of the fuel cell, this value of A must be divided by 2. Thus,

$$A_{\text{fuel cell}} = \frac{0.12\ \text{m}^2}{2}\left(\frac{\underline{\hspace{5em}}\ \text{cm}^2}{1\ \text{m}^2}\right)$$

$$A_{\text{fuel cell}} = \underline{\hspace{4em}}\ \text{cm}^2$$

The dimensions of a fuel cell with a square surface could be obtained as follows:

$$L = \sqrt{A_{\text{fuel cell}}} = \sqrt{\underline{\hspace{4em}}\ \text{cm}^2}$$

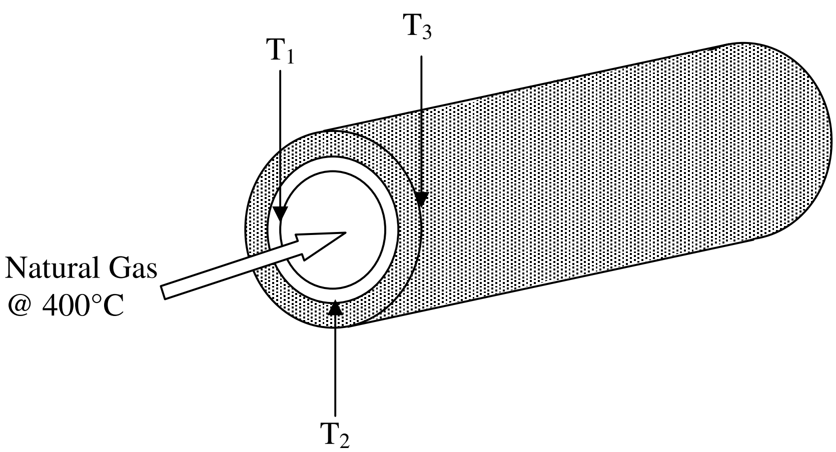
$$\boxed{L = \underline{\hspace{4em}}\ \text{cm}}$$

Therefore, for a heat transfer rate of 183.6 W, air at 25 °C can be used to keep the surface area of a ________ cm x ________ cm fuel cell at a temperature of ________ °C.

### Example 4.3-2: Heat Loss in Fuel Reforming Applications

A pipe made of 308 stainless steel (schedule number 80) with a nominal diameter of 1.5" is carrying methane at a temperature of 400°C in a steam-methane reforming process for producing hydrogen. The pipe is insulated with a layer of glass-fiber with a thickness of 1". Determine the temperature at the interface between the pipe and the glass fiber and the heat loss through the insulated pipe with a length of 15 m. The surface of the insulating material is at a temperature of 25°C.

A schematic of the pipe is shown below:




### Strategy

The equation for the heat loss through a pipe can be applied to the different layers in the pipe.

### Solution

The heat loss through the walls of a cylinder is given by:

$$q = \frac{T_{in} - T_{out}}{R}$$

where:

$T_{in}$ = temperature at the inner wall of the pipe, K

$T_{out}$ = temperature at the outer wall of the pipe, K

R = resistance of the pipe to the heat transfer through its walls, $\frac{K}{W}$

In this problem, we need to apply this equation for both the steel pipe and the insulated pipe. The overall heat loss will be obtained using this equation for the insulated steel pipe. The resistance to heat transfer in cylindrical coordinates is calculated with the following equation:

$$R = \frac{r_{out} - r_{in}}{kA_{lm}}$$

in this equation:

$r_{out}$ = outer radius of the cylinder, m

$r_{in}$ = inner radius of the cylinder, m

k = thermal conductivity of the material, $\frac{W}{m \cdot K}$

$A_{lm}$ = log mean area, $m^2$

The log mean area of the pipe is defined as:

$$A_{lm} = \frac{A_{out} - A_{in}}{\ln\left(\frac{\qquad}{\qquad}\right)}$$

where $A_{out}$ and $A_{in}$ are the outer and inner surface areas of the cylinder, respectively.

Applying the equations for resistance and the log mean areas to the steel and the overall pipe we have:

*Steel Pipe*

$$\dot{q}_{1\to2} = \frac{T_1 - T_2}{R_{1\to2}}$$

$$R_{1\to2} = \frac{r_2 - r_1}{k_{steel} A_{lm,1\to2}}$$

$$A_{lm,1\to2} = \frac{A_2 - A_1}{\ln\left(\frac{A_2}{A_1}\right)}$$

*Overall*

$$\dot{q}_{1\to3} = \frac{T_1 - T_3}{R_{1\to2} + R_{2\to3}}$$

$$A_{lm,1\to3} = \frac{A_3 - A_1}{\ln\left(\frac{A_3}{A_1}\right)}$$

To determine the log mean areas we need to look for the radius of the steel pipe in Appendix A.5 of Geankoplis. For the 1.5" pipe:

$r_{in} = r_1 = \_\_\_\_\_\_\_\_\_\_$ m

$r_{out} = r_2 = \_\_\_\_\_\_\_\_\_\_$ m

The radius of the pipe including the insulation is obtained by adding the thickness of 1" to the outer radius of the steel pipe. Hence,

$r_3 = \_\_\_\_\_\_\_\_\_\_ \text{ m} + 0.0254 \text{ m} = \_\_\_\_\_\_\_\_\_\_ \text{ m}$

With these diameter values and the length of the pipe, the areas $A_1$, $A_2$ and $A_3$ can be calculated as shown in the following steps.

$$A_1 = 2\pi r_1 L = 2\pi(0.01905 \text{ m})(\_\_\_\_\_\_\_\_\_\_ \text{ m}) = 1.795 \text{ m}^2$$

$$A_2 = 2\pi r_2 L = 2\pi(\_\_\_\_\_\_\_\_\_\_ \text{ m})(\_\_\_\_\_\_\_\_\_\_ \text{ m}) = \_\_\_\_\_\_\_\_\_\_ \text{ m}^2$$

$$A_3 = 2\pi r_3 L = 2\pi(\_\_\_\_\_\_\_\_\_\_ \text{ m})(\_\_\_\_\_\_\_\_\_\_ \text{ m}) = \_\_\_\_\_\_\_\_\_\_ \text{ m}^2$$

The thermal conductivities for the glass fiber and the steel can be found in Appendix A.3 of Geankoplis and are shown below. The conductivity of the glass fiber was selected at the highest temperature available in Table A.3-15. The thermal conductivity of steel was obtained from Table A.3-16.

$$k_{\text{steel}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$$

$$k_{\text{glass fiber}} = \_\_\_\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$$

Substituting the values we obtained into the equations for the individual layers yields:

*Steel Pipe*

$$\dot{q}_{1\to2} = \frac{T_1 - T_2}{R_{1\to2}} = \frac{\_\_\_\_\_\_\_\_\_\_ \,{}^\circ\text{C} - T_2}{\_\_\_\_\_\_\_\_\_\_ \,\frac{{}^\circ\text{C}}{\text{W}}}$$

$$R_{1\to2} = \frac{\_\_\_\_\_\_\_\_\_\_ \text{ m} - \_\_\_\_\_\_\_\_\_\_ \text{ m}}{21.6 \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}\left(\_\_\_\_\_\_\_\_\_\_ \text{ m}^2\right)} = 1.17\times10^{-4} \frac{{}^\circ\text{C}}{\text{W}}$$

$$A_{lm,1\to2} = \frac{\_\_\_\_\_\_\_\_\_\_ \text{ m}^2 - \_\_\_\_\_\_\_\_\_\_ \text{ m}^2}{\ln\left(\frac{2.274 \text{ m}^2}{1.795 \text{ m}^2}\right)} = 2.025 \text{ m}^2$$

*Overall*

$$\dot{q}_{1\to 3} = \frac{\_\_\_\_\_\_\_\_\_\_\ {}^\circ\text{C} - \_\_\_\_\_\_\_\_\_\_\ {}^\circ\text{C}}{\_\_\_\_\_\_\_\_\_\_\ \frac{{}^\circ\text{C}}{\text{W}} + 0.139\,\frac{{}^\circ\text{C}}{\text{W}}} = 2695.6\ \text{W}$$

$$R_{2\to 3} = \frac{\_\_\_\_\_\_\_\_\_\_\ \text{m} - 0.02415\ \text{m}}{\_\_\_\_\_\_\_\_\_\_\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}\left(\_\_\_\_\_\_\_\_\_\_\ \text{m}^2\right)} = 0.139\,\frac{{}^\circ\text{C}}{\text{W}}$$

$$A_{lm,2\to 3} = \frac{\_\_\_\_\_\_\_\_\_\_\ \text{m}^2 - \_\_\_\_\_\_\_\_\_\_\ \text{m}^2}{\ln\left(\frac{4.670\ \text{m}^2}{2.274\ \text{m}^2}\right)} = \_\_\_\_\_\_\_\_\_\_\ \text{m}^2$$

Hence, the heat loss through the insulated pipe is 2695.6 W.

Since this answer represents the amount of heat lost per unit time, if we assume that the system is at steady state. The heat loss per unit time will be the same in the individual layers. Thus, we can use the equation for the heat loss through the steel pipe to determine the temperature at the steel-glass fiber interface.

$$\dot{q}_{1\to 2} = \frac{\_\_\_\_\_\_\ {}^\circ\text{C} - T_2}{\_\_\_\_\_\_\_\_\_\_\ \frac{{}^\circ\text{C}}{\text{W}}} = 2695.6\ \text{W}$$

Solving for the temperature $T_2$, we get:

$$T_2 = \_\_\_\_\_\_\_\_\_\_\ {}^\circ\text{C} - 2695.6\ \text{W}\left(\_\_\_\_\_\_\_\_\_\_\_\_\ \frac{{}^\circ\text{C}}{\text{W}}\right)$$

$$\boxed{T_2 = \_\_\_\_\_\_\_\_\_\_\ {}^\circ\text{C}}$$

As it can be seen, the temperature at the pipe - insulator interface is almost the same as the temperature of the inner wall of the steel pipe. This is because most of the heat is lost through the metal pipe due to the high thermal conductivity of steel in comparison to the thermal conductivity of the insulating material.

## Example 4.3-3: Heat Loss by Convection and Conduction in a Steam-Methane Reforming Process

Natural gas at 400°C is flowing inside a steel pipe with an inner diameter of 1.5 in and an outer diameter of 1.9 in. The pipe is insulated with a layer of glass-fiber with a thickness of 1 in. The convective coefficient outside the insulated pipe is $1.23\dfrac{\text{btu}}{\text{ft}^2\cdot\text{hr}\cdot°\text{F}}$. The temperature on the external surface of the pipe is 43.4°C.

Calculate the convective coefficient of natural gas and the overall heat transfer coefficient U based on the inside area $A_i$, if heat is being lost at a rate of $7115\dfrac{\text{btu}}{\text{hr}}$ in a pipe with a length of 49.2 ft.

### Strategy

To determine the heat transfer coefficients, we will use the equation for heat loss for a multilayer cylinder.

### Solution

The heat loss through a cylinder with different layers is defined by the following equation:

$$\dot{q} = \frac{T_i - T_o}{\sum R} = \frac{T_i - T_o}{R_i + R_A + R_B + R_o}$$

where:

$T_i$ = Temperature on the internal surface of the pipe

$T_o$ = Temperature on the external surface of the pipe

$R_i$ = Convective resistance inside the pipe

$R_A$ = Conductive resistance through the steel pipe

$R_B$ = Conductive resistance through the insulation layer

$R_o$ = Convective resistance outside the pipe

The resistance to heat transfer due to convection is defined as follows:

$$R_{conv} = \frac{1}{hA}$$

where:

$h$ = Convective heat transfer coefficient

$A$ = Area of heat transfer

The resistance of a cylinder to heat conduction is calculated as follows:

$$R_{cond} = \frac{r_{out} - r_{in}}{kA_{lm}}$$

where:

$r_{out}$ = Outer radius of the cylinder

$r_{in}$ = Inner radius of the cylinder

$k$ = thermal conductivity of the material

$A_{lm}$ = log mean area of the cylinder

We can enter the definitions of the resistances due to conduction and convection into the equation for the heat loss to yield:

$$\dot{q} = \frac{T_i - T_o}{\frac{1}{h_i A_i} + \frac{r_1 - r_i}{k_{steel} A_{A,lm}} + \frac{r_o - r_1}{k_{glass-fiber} A_{B,lm}} + \frac{1}{h_o A_o}}$$

In this equation:

$r_i$ = inner radius of the steel pipe = $\frac{1.5 \text{ in}}{2}$ = ____________ in

$r_1$ = outer radius of the steel pipe = $\frac{1.9 \text{ in}}{2}$ = ____________ in

$r_o$ = outer radius of the insulated pipe = ___________ in

With these values we can calculate the log mean areas $A_{A,lm}$ and $A_{B,lm}$ and the inner and outer areas of the insulated pipe. Thus,

$$A_i = 2\pi r_i L = 2\pi (0.75 \text{ in}) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) (\_\_\_\_\_\_\_\_\_\_\_ \text{ ft}) = \_\_\_\_\_\_\_\_\_\_\_ \text{ft}^2$$

$$A_1 = 2\pi r_1 L = 2\pi (\_\_\_\_\_\_\_\_\_\_\_ \text{ in}) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) (\_\_\_\_\_\_\_\_\_\_\_ \text{ ft}) = 24.48 \text{ ft}^2$$

$$A_o = 2\pi r_o L = 2\pi(\_\_\_\_\_\_\_\_\_\_\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)(\_\_\_\_\_\_\_\_\_\_\ \text{ft}) = \_\_\_\_\_\_\_\_\_\_\ \text{ft}^2$$

$$A_{A,lm} = \frac{A_1 - A_i}{\ln\left(\dfrac{A_1}{A_i}\right)} = \frac{24.48\ \text{ft}^2 - \_\_\_\_\_\_\_\_\_\_\ \text{ft}^2}{\ln\left(\dfrac{24.48\ \text{ft}^2}{\_\_\_\_\_\_\_\_\_\_\ \text{ft}^2}\right)} = 21.80\ \text{ft}^2$$

$$A_{B,lm} = \frac{A_o - A_1}{\ln\left(\dfrac{A_o}{A_1}\right)} = \frac{\_\_\_\_\_\_\_\_\_\_\ \text{ft}^2 - 24.48\ \text{ft}^2}{\ln\left(\dfrac{\_\_\_\_\_\_\_\_\_\_\ \text{ft}^2}{24.48\ \text{ft}^2}\right)} = \_\_\_\_\_\_\_\_\_\_\ \text{ft}^2$$

The temperature of the inner and outer surfaces of the pipe are given in °C and therefore, they must be converted to °F:

$$T_i(°\text{F}) = (400°\text{C} \times 1.8) + 32 = \_\_\_\_\_\_\_\_\_\_\ °\text{F}$$

$$T_o(°\text{F}) = (43.4°\text{C} \times 1.8) + 32 = \_\_\_\_\_\_\_\_\_\_\ °\text{F}$$

Now we can substitute all the values into the heat loss equation and solve for the convective coefficient $h_i$:

$$h_i = \frac{1}{A_i}\left[\frac{T_i - T_o}{\dot{q}} - \frac{r_1 - r_i}{k_{steel} A_{A,lm}} - \frac{r_o - r_1}{k_{glass-fiber} A_{B,lm}} - \frac{1}{h_o A_o}\right]^{-1}$$

The thermal conductivity values can be obtained from Appendices A.3-15 and A.3-16 of Geankoplis. However, since the values are given in the SI system they must be converted to the English system. Hence,

$$k_{steel} = 45 \frac{\text{J}}{\text{m} \cdot \text{s} \cdot °\text{C}}\left(\frac{1\ \text{btu}}{\_\_\_\_\_\_\_\_\_\_\ \text{J}}\right)\left(\frac{\_\_\_\_\_\_\_\_\_\_\ \text{m}}{1\ \text{ft}}\right)\left(\frac{3600\ \text{s}}{1\ \text{hr}}\right)\left(\frac{1\ °\text{C}}{\_\_\_\_\_\_\_\_\_\_\ °\text{F}}\right) = 26 \frac{\text{btu}}{\text{ft} \cdot \text{hr} \cdot °\text{F}}$$

$$k_{glass-fiber} = \_\_\_\_\_\_ \frac{\text{J}}{\text{m} \cdot \text{s} \cdot °\text{C}}\left(\frac{1\ \text{btu}}{\_\_\_\_\_\_\_\_\_\_\ \text{J}}\right)\left(\frac{\_\_\_\_\_\_\_\_\_\_\ \text{m}}{1\ \text{ft}}\right)\left(\frac{3600\ \text{s}}{1\ \text{hr}}\right)\left(\frac{1\ °\text{C}}{\_\_\_\_\_\ °\text{F}}\right) = 0.0317 \frac{\text{btu}}{\text{ft} \cdot \text{hr} \cdot °\text{F}}$$

The thermal conductivities of steel and glass-fiber were obtained at the highest temperature available in Appendix A.3.

Now we can calculate the convective heat transfer coefficient $h_i$ as shown below:

$$h_i = \frac{1}{\_\_\_\_\_\_\ \text{ft}^2}\left[\frac{\_\_\_\_\_\_\ ^\circ\text{F} - 110.1^\circ\text{F}}{7115\dfrac{\text{btu}}{\text{hr}}} - \frac{(\_\_\_\_\_\_\ \text{in} - 0.75\ \text{in})\left(\dfrac{1\ \text{ft}}{\_\_\_\_\_\_\ \text{in}}\right)}{26\dfrac{\text{btu}}{\text{ft}\cdot\text{hr}\cdot^\circ\text{F}}(\_\_\_\_\_\_\ \text{ft}^2)} - \frac{(\_\_\_\_\_\_\ \text{in} - 0.95\ \text{in})\left(\dfrac{1\ \text{ft}}{\_\_\_\_\_\_\ \text{in}}\right)}{0.0317\dfrac{\text{btu}}{\text{ft}\cdot\text{hr}\cdot^\circ\text{F}}(\_\_\_\_\_\_\ \text{ft}^2)} - \frac{1}{\_\_\_\_\_\_\ \dfrac{\text{btu}}{\text{ft}^2\cdot\text{hr}\cdot^\circ\text{F}}(\_\_\_\_\_\_\ \text{ft}^2)}\right]^{-1}$$

$$h_i = \frac{1}{\_\_\_\_\_\_\ \text{ft}^2}\left[\_\_\_\_\_\_\ \frac{\text{hr}\cdot^\circ\text{F}}{\text{btu}} - 2.95\times10^{-5}\frac{\text{hr}\cdot^\circ\text{F}}{\text{btu}} - \_\_\_\_\_\_\ \frac{\text{hr}\cdot^\circ\text{F}}{\text{btu}} - 0.0162\frac{\text{hr}\cdot^\circ\text{F}}{\text{btu}}\right]^{-1}$$

$$\boxed{h_i = \_\_\_\_\_\_\ \frac{\text{btu}}{\text{ft}^2\cdot\text{hr}\cdot^\circ\text{F}}}$$

To calculate the overall heat transfer coefficient U we need to use the equation for the heat loss in terms of U. Thus,

$$\dot{q} = U_i A_i (T_i - T_o)$$

We can solve this equation for the coefficient U and substitute the corresponding values to get:

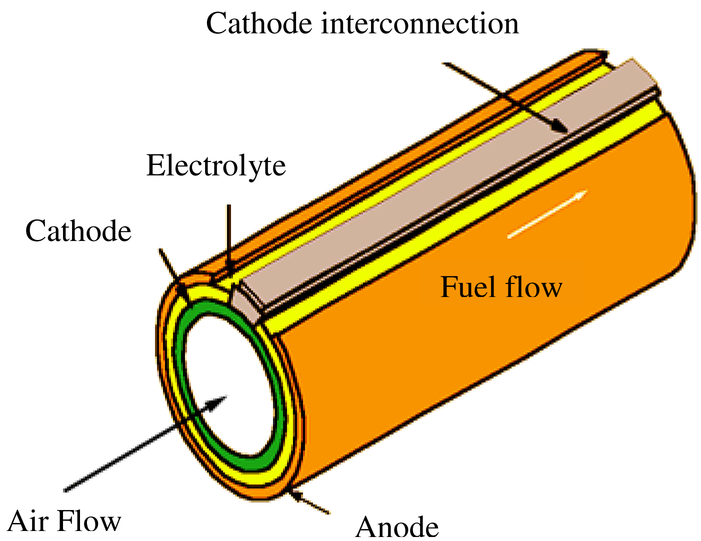
$$U_i = \frac{\dot{q}}{A_i(T_i - T_o)} = \frac{7115\dfrac{\text{btu}}{\text{hr}}}{\_\_\_\_\_\_\ \text{ft}^2(\_\_\_\_\_\_\ ^\circ\text{F} - 110.1^\circ\text{F})}$$

$$\boxed{U_i = \_\_\_\_\_\_\ \frac{\text{btu}}{\text{ft}^2\cdot\text{hr}\cdot^\circ\text{F}}}$$

### Example 4.3-4: Heat Generation in a Tubular Solid-Oxide Fuel Cell

A tubular solid-oxide fuel cell with an outer diameter of 2.2 cm and a length of 150 cm is operating at a current density of $202.6\frac{\text{mA}}{\text{cm}^2}$. Determine the heat generation rate in $\frac{\text{W}}{\text{m}^3}$ if the voltage of the fuel cell is 1 V. Assume that the thickness of the electrodes and electrolyte membrane are small compared to the overall diameter of the fuel cell.

The following figure shows a tubular solid-oxide fuel cell:




### Strategy

The heat generation rate of the fuel cell can be obtained from the power of the fuel cell, which depends on the current and the voltage.

### Solution

The heat generated by the fuel cell in terms of the power is given by the equation shown below:

$$\dot{q} = \frac{P}{\pi R^2 L}$$

The power of the fuel cell can be obtained by multiplying the current by the voltage of the fuel cell. Hence,

$$P = IV$$

Substituting this equation into the equation for the heat generation rate yields:

$$\dot{q} = \frac{IV}{\pi R^2 L}$$

The problem statement is not giving the value of the current. However, if we calculate the cross-surface area of the fuel cell we can determine the value of the current in A:

$$i = \frac{I}{A_{surface}} = \frac{I}{2\pi RL}$$

Solving for the current I and substituting the dimensions of the fuel cell into this equation, we get:

$$I = \pi DLi = \pi(\underline{\hspace{2cm}}\ \text{cm})(\underline{\hspace{2cm}}\ \text{cm})\left(202.6\frac{\text{mA}}{\text{cm}^2}\right)\left(\frac{1\ \text{A}}{1000\ \text{mA}}\right)$$

$$I = \underline{\hspace{2cm}}\ \text{A}$$

Entering this value into the heat generation equation we have:

$$\dot{q} = \frac{\underline{\hspace{2cm}}\ \text{A}(1\ \text{V})}{\pi\left(\frac{\underline{\hspace{2cm}}\ \text{cm}}{2}\right)^2(\underline{\hspace{2cm}}\ \text{cm})\left(\frac{1\ \text{m}^3}{\underline{\hspace{2.5cm}}\ \text{cm}^3}\right)}$$

$$\boxed{\dot{q} = 3.68\times10^5\ \frac{\text{W}}{\text{m}^3}}$$

### Example 4.5-1: Heating of Natural Gas- in Steam-Methane Reforming Process

Natural gas at a temperature of 310°C is flowing inside a steel pipe Schedule 80 with an inner diameter of 1.5 in at a rate of $7.79\times10^{-3}\ \frac{m^3}{s}$. The natural gas is being heated by the product of the reforming process at 850°C. The convective heat transfer coefficient of the reformate is $1025\frac{W}{m^2\cdot K}$. Calculate the heat transfer rate in $W$ through a pipe with a length of 7 m. The properties of natural gas are given in the following table.

| | |
|---|---|
| $\rho$ | $7.859\frac{kg}{m^3}$ |
| $C_p$ | $3087\frac{J}{kg\cdot K}$ |
| k | $0.0803\frac{W}{m\cdot K}$ |
| $\mu_b$ | $1.942\times10^{-5}\ \frac{kg}{m\cdot s}$ |
| $\mu_w$ | $2.909\times10^{-5}\ \frac{kg}{m\cdot s}$ |

### Strategy

The equation for heat transfer through a pipe will be used to determine the heat flux.

### Solution

When heat is being transferred through a fluid, the heat flux is given by:

$$\dot{q} = \frac{(T_r - T_n)}{\sum R}$$

where:

$\dot{q}$ = Heat transfer rate in W

$T_r$ = Temperature of the heating medium (reformate), °C or K

$T_n$ = Temperature of the fluid inside the pipe (natural gas), °C or K

$\sum R$ = Sum of resistances to heat transfer through the pipe, $\frac{°C}{W}$ or $\frac{K}{W}$

In this problem, the sum of the resistances is given by the sum of two convective resistances (fluid inside and outside the pipe) and the resistance to heat flow through the steel pipe. Thus,

$$\sum R = \frac{1}{h_i A_i} + \frac{r_o - r_i}{k_{steel} A_{lm}} + \frac{1}{h_o A_o}$$

The parameters we need to calculate before being able to use the equation for heat transfer in terms of the resistances are: $A_i$, $A_{lm}$, $A_o$ and $h_i$. The heat transfer areas are calculated using the inner and outer diameters of the pipe from Appendix A.5.

$$A_i = \pi D_i L = \pi(0.0381\text{ m})(7\text{ m}) = \_\_\_\_\_\_\_\_\_\ \text{m}^2$$

$$A_o = \pi D_o L = \pi(\_\_\_\_\_\_\_\_\_\ \text{m})(7\text{ m}) = \_\_\_\_\_\_\_\_\_\ \text{m}^2$$

$$A_{lm} = \frac{A_o - A_i}{\ln\left(\frac{A_o}{A_i}\right)} = \frac{\_\_\_\_\_\_\_\_\_\ \text{m}^2 - \_\_\_\_\_\_\_\_\_\ \text{m}^2}{\ln\left(\frac{\_\_\_\_\_\_\_\_\_\ \text{m}^2}{\_\_\_\_\_\_\_\_\_\ \text{m}^2}\right)} = 0.945\text{ m}^2$$

The following correlation can be used for calculating the heat transfer coefficient for an aspect ratio (length/diameter) of the pipe greater than 60.

$$h_L = 0.027\frac{k}{D} N_{Re}^{\ 0.8} N_{Pr}^{\ 1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

where:

$k$ = Thermal conductivity of the fluid inside the pipe, $\frac{\text{W}}{\text{m}\cdot\text{K}}$

$N_{Re}$ = Reynolds number

$N_{Pr}$ = Prandtl number

$\mu_b$ = Viscosity of the fluid in the pipe at the bulk temperature, $\frac{\text{kg}}{\text{m}\cdot\text{s}}$

$\mu_w$ = Viscosity of the fluid in the pipe at the temperature of the inner wall, $\frac{\text{kg}}{\text{m}\cdot\text{s}}$

In this problem, $\frac{L}{D} = \frac{7\text{ m}}{0.0381\text{ m}} = \_\_\_\_\_\_\_\_\_$. Hence, we can use this correlation to calculate $h_L$.

The dimensionless quantities $N_{Re}$ and $N_{Pr}$ are defined as follows:

$$N_{Re} = \frac{D\upsilon\rho}{\mu} \qquad\qquad N_{Pr} = \frac{C_p\mu}{k}$$

The velocity of the natural gas is obtained by dividing the volumetric flow rate by the cross-sectional area of the pipe. Thus, after substituting the corresponding quantities into the equations for Reynolds and Prandtl numbers, we have:

$$N_{Re} = \frac{D\left(\frac{\dot{V}}{\pi r_i^2}\right)\rho}{\mu} = \frac{0.0381\ \text{m}\left(\frac{\underline{\qquad\qquad}\ \frac{\text{m}^3}{\text{s}}}{\underline{\qquad\qquad}}\right)7.859\ \frac{\text{kg}}{\text{m}^3}}{\underline{\qquad\qquad}\ \frac{\text{kg}}{\text{m}\cdot\text{s}}} = \underline{\qquad\qquad}$$

$$N_{Pr} = \frac{C_p\mu}{k} = \frac{3087\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\left(\underline{\qquad\qquad}\ \frac{\text{kg}}{\text{m}\cdot\text{s}}\right)}{\underline{\qquad\qquad}\ \frac{\text{W}}{\text{m}\cdot\text{K}}} = \underline{\qquad\qquad}$$

Now we can substitute the dimensionless numbers we just calculated and the properties of the fluid into the equation for the heat transfer coefficient to yield:

$$h_L = 0.027\frac{0.0803\ \frac{\text{W}}{\text{m}\cdot\text{K}}}{0.0381\ \text{m}}\left(\underline{\qquad\qquad}\right)^{0.8}\left(\underline{\qquad}\right)^{1/3}\left(\frac{\underline{\qquad\qquad}\ \frac{\text{kg}}{\text{m}\cdot\text{s}}}{\underline{\qquad\qquad}\ \frac{\text{kg}}{\text{m}\cdot\text{s}}}\right)^{0.14}$$

$$h_L = \underline{\qquad\qquad}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}$$

The convective heat transfer coefficient $h_L$ was obtained using the properties of the fluid inside the pipe. Therefore, the heat transfer coefficient $h_L$ is equal to the heat transfer coefficient $h_i$. Substituting the corresponding quantities into the equation for the heat transfer rate $\dot{q}$. The thermal conductivity was obtained from Appendix A.3.

$$\dot{q} = \frac{(850°\text{C} - 310°\text{C})}{\frac{1}{\underline{\qquad}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(\underline{\qquad}\ \text{m}^2\right)} + \frac{0.04826\ \text{m} - 0.0381\ \text{m}}{45\ \frac{\text{W}}{\text{m}\cdot\text{K}}\left(0.945\ \text{m}^2\right)} + \frac{1}{\underline{\qquad}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(\underline{\qquad}\ \text{m}^2\right)}}$$

$$\boxed{\dot{q} = \underline{\qquad\qquad}\ \text{W}}$$

### Example 4.5-2: Trial-and-Error Solution for Heating of Steam

Steam at a temperature of 150°C is being heated before entering a steam-methane reforming unit to produce hydrogen for fuel cells. The heating medium is the synthesis gas produced by the steam-methane reforming unit at 850 °C. The convective heat transfer coefficient of the syngas is $950\dfrac{\mathrm{W}}{\mathrm{m^2\cdot K}}$. The steam is flowing in a 1.5-in schedule 40 steel pipe at a velocity of $20.1\dfrac{\mathrm{m}}{\mathrm{s}}$. Determine the overall coefficient $U_i$ for a pipe with a length of 6.2 m.

### Strategy

We can determine the convective heat transfer coefficient using the properties of steam at the temperature of the inner wall of the pipe. This temperature will be determined by trial and error.

### Solution

The overall heat transfer coefficient can be determined from the equation for the heat transfer rate through the pipe:

$$\dot{q} = U_i A_i (T_o - T_i) = \frac{T_o - T_i}{\sum R}$$

The temperature difference $T_o$ - $T_i$ can be eliminated from this equation to yield:

$$U_i = \frac{1}{\underline{\hspace{3cm}}}$$

where:

$$\sum R = \frac{1}{h_i A_i} + \frac{r_o - r_i}{k A_{lm}} + \frac{1}{h_o A_o}$$

The convective coefficient $h_i$ can be calculated using the following correlation.

$$h_L = 0.027 \frac{k}{D} N_{Re}{}^{0.8} N_{Pr}{}^{1/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

The dimensionless quantities in this equation will be determined using the properties of steam at the temperature of the inner wall of the pipe. For the first trial, this temperature will be assumed to be about one-quarter the difference between the temperatures of the steam and the air. Thus,

$$T_{w,assumed} = \frac{850°C - 150°C}{4} + 150°C = \underline{\hspace{2.5cm}}\ °C$$

From Table A.2-12 of Geankoplis we can get the properties of steam at a temperature of 148.9 °C, which is relatively close to the bulk temperature of the steam inside the pipe (150°C).

$$\rho = \_\_\_\_\_\_\_\_\_\_ \frac{kg}{m^3} \qquad \mu = \mu_b = 1.488\times10^{-5}\,\frac{kg}{m\cdot s} \qquad N_{Pr} = \_\_\_\_\_\_\_\_\_\_$$

$$C_p = 1909\,\frac{J}{kg\cdot K} \qquad k = \_\_\_\_\_\_\_\_\_\_ \frac{W}{m\cdot K}$$

The other parameter required to use the correlation for calculating $h_L$ is the viscosity of steam at $T_w$. This can be obtained using linear interpolation from the data in Table A.2-12. Hence,

$$\frac{325°C - 315.6°C}{371.1°C - 315.6°C} = \frac{\mu_w - 2.113\times10^{-5}\,\frac{kg}{m\cdot s}}{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{kg}{m\cdot s} - 2.113\times10^{-5}\,\frac{kg}{m\cdot s}}$$

Solving for the viscosity $\mu_w$ we get:

$$\mu_w = \frac{325°C - 315.6°C}{371.1°C - 315.6°C}\left(\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{kg}{m\cdot s} - 2.113\times10^{-5}\,\frac{kg}{m\cdot s}\right) + \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{kg}{m\cdot s}$$

$$\mu_w = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{kg}{m\cdot s}$$

With the properties of steam, we can now determine the Reynolds number as shown in the following steps. The diameter of the pipe was obtained from Table A.5-1 of Geankoplis

$$N_{Re} = \frac{D\upsilon\rho}{\mu} = \frac{\_\_\_\_\_\_\_\_\_\_\ m\left(\_\_\_\_\_\_\_\ \frac{m}{s}\right)\left(0.525\,\frac{kg}{m^3}\right)}{1.488\times10^{-5}\,\frac{kg}{m\cdot s}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Substituting this value into the correlation for $h_L$ we get:

$$h_L = 0.027\,\frac{\_\_\_\_\_\_\_\_\_\_\_\_\ \frac{W}{m\cdot K}}{0.04089\ m}\left(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\right)^{0.8}(0.95)^{1/3}\left(\frac{1.488\times10^{-5}\,\frac{kg}{m\cdot s}}{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{kg}{m\cdot s}}\right)^{0.14}$$

$$h_L = \_\_\_\_\_\_\_\_\_\_\_\_ \frac{W}{m^2\cdot K}$$

Now we can proceed to calculate the sum of the resistances to heat transfer as follows:

$$\sum R = \frac{1}{h_i A_i} + \frac{r_o - r_i}{k A_{lm}} + \frac{1}{h_o A_o}$$

The heat transfer areas are calculated using the inner and outer diameters of the pipe from Appendix A.5.

$$A_i = \pi D_i L = \pi (0.04089 \text{ m})(\_\_\_\_\_\_ \text{ m}) = \_\_\_\_\_\_\_\_\_\_\_ \text{ m}^2$$

$$A_o = \pi D_o L = \pi (0.04826 \text{ m})(\_\_\_\_\_\_ \text{ m}) = \_\_\_\_\_\_\_\_\_\_\_ \text{ m}^2$$

$$A_{lm} = \frac{A_o - A_i}{\ln\left(\frac{A_o}{A_i}\right)} = \frac{\_\_\_\_\_\_\_ \text{ m}^2 - \_\_\_\_\_\_\_ \text{ m}^2}{\ln\left(\frac{\_\_\_\_\_\_\_ \text{ m}^2}{\_\_\_\_\_\_\_ \text{ m}^2}\right)} = 0.866 \text{ m}^2$$

These values can be entered into the equation for the sum of resistances to yield:

$$\sum R = \frac{1}{\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}^2 \cdot \text{K}}\left(\_\_\_\_\_\_\_\_\_ \text{ m}^2\right)} + \frac{0.04826 \text{ m} - 0.04089 \text{ m}}{45 \frac{\text{W}}{\text{m} \cdot \text{K}}\left(0.866 \text{ m}^2\right)} + \frac{1}{950 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}\left(\_\_\_\_\_\_\_ \text{ m}^2\right)}$$

$$\sum R = \_\_\_\_\_\_\_\_\_ \frac{\text{K}}{\text{W}}$$

The thermal conductivity value of steel was obtained from Table A.3-16. To determine if the value of $T_w$ we selected is correct, we need to solve for the temperature from the equation for the heat resistance due to the steam in the pipe:

$$T_{w,calculated} - T_b = \frac{R_i}{\sum R}\left(T_o - T_b\right)$$

Solving for $T_w$ and substituting the rest of the values into this equation, we have that:

$$T_{w,calculated} = T_b + \frac{1}{h_i A_i \sum R}\left(T_o - T_b\right)$$

$$T_{w,calculated} = 150°\text{C} + \frac{1}{\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}^2 \cdot \text{K}}\left(\_\_\_\_\_\_\_ \text{ m}^2\right)\left(\_\_\_\_\_\_\_ \frac{\text{K}}{\text{W}}\right)}\left(850°\text{C} - 150°\text{C}\right)$$

$$T_{w,calculated} = \_\_\_\_\_\_\_\_\_ °\text{C}$$

It can be see that the $T_{w,assumed}$ does not match $T_{w,calculated}$. Hence, we have to repeat the procedure for determining the temperature of the inner wall. The $T_w$ value will affect the value of $h_L$ since we have to look for a new value of $\mu_w$ in Appendix A.3.

Selecting a higher $T_w$ for the second trial will yield a higher $\mu_w$, resulting in a lower heat transfer coefficient $h_L$ and a higher $\sum R$. For the second trial, we will select:

$T_{w,assumed} = 800°C$

Substituting the viscosity $\mu_w$ into the equation for the heat transfer coefficient $h_L$ yields:

$$h_L = 0.027 \frac{\underline{\qquad\qquad} \frac{W}{m \cdot K}}{0.04089\ m} (\underline{\qquad\qquad\qquad})^{0.8} (\underline{\qquad})^{1/3} \left( \frac{1.488 \times 10^{-5} \frac{kg}{m \cdot s}}{3.95 \times 10^{-5} \frac{kg}{m \cdot s}} \right)^{0.14}$$

$$h_L = \underline{\qquad\qquad} \frac{W}{m^2 \cdot K}$$

For this value of $h_L$ we will get the sum of the resistances as follows:

$$\sum R = \frac{1}{62.3 \frac{W}{m^2 \cdot K} (\underline{\qquad\qquad}\ m^2)} + \frac{0.04826\,m - 0.04089\ m}{\underline{\qquad\qquad} \frac{W}{m \cdot K} (0.866\ m^2)} + \frac{1}{950 \frac{W}{m^2 \cdot K} (\underline{\qquad\qquad}\ m^2)}$$

$$\sum R = \underline{\qquad\qquad} \frac{K}{W}$$

Solving for $T_w$ and substituting the rest of the values into this equation, we have that:

$$T_{w,calculated} = T_b + \frac{1}{h_i A_i \sum R}(T_o - T_b) = 150°C + \frac{1}{62.3 \frac{W}{m^2 \cdot K} (\underline{\qquad\qquad}\ m^2) \left( \underline{\qquad\qquad} \frac{K}{W} \right)} (850°C - 150°C)$$

$$T_{w,calculated} = \underline{\qquad\qquad\qquad} °C$$

The only property that will change for the third trial is the viscosity $\mu_w$. By changing the temperature again, the effect on the convective coefficient $h_L$ will be negligible. Hence, we can use the calculated $T_w$ value of ________°C.

Now we can substitute the values of $\sum R$ at $T_w$ = _________°C and the inner area of the pipe to obtain the overall heat transfer coefficient $U_i$ as shown in the following steps:

$$U_i = \frac{1}{A_i \sum R}$$

$$U_i = \frac{1}{\left(\_\_\_\_\_\_\_\_\ \text{m}^2\right)\left(\_\_\_\_\_\_\_\_\ \frac{\text{K}}{\text{W}}\right)}$$

$$\boxed{U_i = 59.8\frac{\text{W}}{\text{m}^2\cdot\text{K}}}$$

### Example 4.5-3: Heating of Ethanol in Reforming Process

A vapor mixture of ethanol and water is used in a reforming process to produce hydrogen for proton-exchange membrane fuel cells. In a distributed-scale process, the ethanol mixture is flowing at a rate of $0.367\frac{\text{kg}}{\text{s}}$ and is entering a 1" steel pipe Schedule 40 at a temperature of 210.4°C. Determine the length of the pipe required if the vapor is exiting at 350°C and the inner wall of the pipe is at a constant temperature of 270°C. The properties of the vapor mixture are summarized in the following table.

| | |
|---|---|
| $\mu$ | $1.284\times10^{-5}\,\text{Pa}\cdot\text{s}$ |
| $\rho$ | $13.2\frac{\text{kg}}{\text{m}^3}$ |
| $C_p$ | $2211\frac{\text{J}}{\text{kg}\cdot\text{K}}$ |
| k | $0.03631\frac{\text{W}}{\text{m}\cdot\text{K}}$ |

### Strategy

To determine the heat transfer area we can use the equation for heat flux through a fluid.

### Solution

The heat flux when heat is being transferred by a fluid is given by:

$$\frac{\dot{q}}{A} = h_L\left(T_w - T\right)$$

where $T_w$ is the temperature of the inner wall of the pipe, and T is the bulk temperature of the ethanol and water mixture. Since the problem statement is giving the flow rate and properties of the fluid, the heat transfer rate can be calculated with the equation for sensible heat:

$$\dot{q} = \dot{m}C_p\left(T_{out} - T_{in}\right)$$

In this equation, $T_{in}$ and $T_{out}$ are the temperatures of the ethanol/water mixture at the inlet and outlet points, respectively.

Substituting this equation into the equation for heat flux, we get:

$$\frac{\dot{m}C_p\left(T_{out} - T_{in}\right)}{\_\_\_\_\_\_} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Solving for the heat transfer area we have:

$$A = \frac{\dot{m}C_p\left(T_{out} - T_{in}\right)}{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

To determine the heat convective heat transfer coefficient, the following correlation can be used for a pie with constant wall temperature and if $N_{Pe}$>100 and L/D>60:

$$h_L = \frac{k}{D}\left(5.0 + 0.025 N_{Pe}^{\ 0.8}\right)$$

where:

$N_{Pe}$ = Peclet number

We can calculate Peclet number by multiplying Reynolds number by Prandtl number and thus determine if it is valid to use this correlation.

$N_{Pe} = N_{Re}N_{Pr}$

$$N_{Re} = \frac{D\upsilon\rho}{\mu}$$

$$N_{Pr} = \frac{C_p\mu}{k} = \frac{2211\frac{J}{kg\cdot K}\left(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\ Pa\cdot s\right)}{0.03631\frac{W}{m\cdot K}} = \_\_\_\_\_\_$$

The velocity of the fluid is calculated by dividing the volumetric flow rate by the cross-sectional area of the pipe. The diameter of the pipe is obtained from Appendix A.5 of Geankoplis.

$$\upsilon = \frac{\dot{m}}{\rho A_{cross}} = \frac{\_\_\_\_\_\_\_\_\ \frac{kg}{s}}{13.2\frac{kg}{m^3}\left[\frac{\pi\left(\_\_\_\_\_\_\_\ m\right)^2}{4}\right]} = \_\_\_\_\_\_\_\_\ \frac{m}{s}$$

Entering this velocity value into the definition of Reynolds number yields:

$$N_{Re} = \frac{D\upsilon\rho}{\mu} = \frac{\_\_\_\_\_\_\_\_\ m\left(\_\_\_\_\_\_\_\_\ \frac{m}{s}\right)\left(13.2\frac{kg}{m^3}\right)}{1.284\times10^{-5}\ \frac{kg}{m\cdot s}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Now we can determine the Peclet number to be given by:

$$N_{Pe} = \underline{\hspace{3cm}}(\underline{\hspace{1.5cm}})$$

$$N_{Pe} = \underline{\hspace{3.5cm}}$$

Therefore, the correlation we selected is valid for this problem and the convective heat transfer coefficient is found to be:

$$h_L = \frac{0.03631\frac{W}{m \cdot K}}{\underline{\hspace{2.5cm}}\ m}\left[5.0 + 0.025\left(\underline{\hspace{4cm}}\right)^{0.8}\right]$$

$$h_L = \underline{\hspace{3cm}}\ \frac{W}{m^2 \cdot K}$$

This value can be entered into the equation for the heat transfer area to yield:

$$A = \frac{\dot{m}C_p\left(T_{out} - T_{in}\right)}{h_L\left(T_w - T\right)} = \frac{0.367\frac{kg}{s}\left(2211\frac{J}{kg \cdot K}\right)\left(\underline{\hspace{1.5cm}}\ ^\circ C - 210.4^\circ C\right)}{\underline{\hspace{2cm}}\ \frac{W}{m^2 \cdot K}\left(\underline{\hspace{1.5cm}}\ ^\circ C - 210.4^\circ C\right)}$$

$$A = \underline{\hspace{2.5cm}}\ m^2$$

The area for heat transfer is given by:

$$A = \pi DL$$

This equation can be solved for the length of the pipe L to give:

$$L = \frac{\underline{\hspace{2cm}}\ m^2}{\pi\left(\underline{\hspace{3cm}}\ m\right)}$$

$$\boxed{L = \underline{\hspace{2cm}}\ m}$$

### Example 4.5-4: Heat-Transfer Area and Log Mean Temperature Difference

An ethanol/water vapor mixture with a heat capacity of $2.23\frac{\text{kJ}}{\text{kg}\cdot°\text{C}}$ in a mid-scale ethanol-reforming plant is heated from 210.4 °C to 350 °C. This mixture is flowing at a rate of $5.18\times10^{5}\frac{\text{kg}}{\text{day}}$. The vapor is being heated by air flowing at a rate of $2.752\times10^{6}\frac{\text{kg}}{\text{day}}$, temperature of 560.6 °C and a heat capacity of $1.166\frac{\text{kJ}}{\text{kg}\cdot°\text{C}}$. What type of flow for this heat exchanger will you select between countercurrent and parallel flow if the overall heat transfer coefficient is $92.4\frac{\text{W}}{\text{m}^2\cdot°\text{C}}$?.

### Strategy

To determine which type of flow is more efficient for this process we need to determine the heat transfer area for both types of flow.

### Solution

The amount of heat gained by the ethanol mixture in terms of the overall heat transfer coefficient is given by the equation shown below:

$$\dot{q}=U_iA_i\Delta T_{lm}$$

where $\Delta T_{lm}$ is the log mean temperature difference defined as:

$$\Delta T_{lm}=\frac{\Delta T_2-\Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

The $\dot{q}$ equation can be solved for the area $A_i$ to yield:

$$A_i=\frac{\dot{q}\rule{2cm}{0.4pt}}{\rule{2cm}{0.4pt}}$$

Since we know the inlet and outlet temperatures of the ethanol/water mixture we can calculate the amount of heat gained as shown in the following steps:

$$\dot{q}=\dot{m}_{ethanol}C_{p,ethanol}\left(T_{ethanol,out}-T_{ethanol,in}\right)$$

$$\dot{q}=5.18\times10^{5}\frac{\text{kg}}{\text{day}}\left(2.23\frac{\text{kJ}}{\text{kg}\cdot°\text{C}}\right)(\_\_\_\_\_\_°\text{C}-\_\_\_\_\_\_°\text{C})\left(\frac{1\text{ day}\rule{1.5cm}{0.4pt}}{\rule{2cm}{0.4pt}}\right)$$

$$\dot{q} = \_\_\_\_\_\_\_\_\_\_ \text{ kW}$$

To determine the log mean temperature difference required to calculate the heat transfer area we need to obtain the temperature of the air leaving the heat exchanger. We know that the amount of heat gained by the ethanol mixture is being lost by the heating air. Thus, the temperature can be determined as follows:

$$\dot{q} = \dot{m}_{air} C_{p,air} \left( T_{air,in} - T_{air,out} \right)$$

$$T_{air,out} = \_\_\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ \text{ °C} - \frac{\_\_\_\_\_\_\_\_\_\_ \text{ kW}}{2.752 \times 10^6 \dfrac{\text{kg}}{\text{day}} \left( 1.166 \dfrac{\text{kJ}}{\text{kg} \cdot \text{°C}} \right) \left( \dfrac{1 \text{ day}}{\_\_\_\_\_\_\_\_\_\_} \right)}$$

$$T_{air,out} = \_\_\_\_\_\_\_\_\_\_ \text{ °C}$$

Now that we know the inlet and outlet temperatures of both the air and the ethanol mixture, we can calculate the log mean temperature difference. Hence,

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln \left( \dfrac{\Delta T_2}{\Delta T_1} \right)}$$

where:

$$\Delta T_{1,countercurrent} = T_{air,in} - T_{ethanol,out} = \_\_\_\_\_\_\_\_\_\_ \text{ °C} - \_\_\_\_\_\_\_\_\_\_ \text{ °C}$$

$$\Delta T_{1,countercurrent} = \_\_\_\_\_\_\_\_\_\_ \text{ °C}$$

$$\Delta T_{2,countercurrent} = T_{air,out} - T_{ethanol,in} = \_\_\_\_\_\_\_\_\_\_ \text{ °C} - \_\_\_\_\_\_\_\_\_\_ \text{ °C}$$

$$\Delta T_{2,countercurrent} = \_\_\_\_\_\_\_\_\_\_ \text{ °C}$$

Substituting the values of $\Delta T_i$ into the definition of the log mean temperature difference we get:

$$\Delta T_{lm,countercurrent} = \frac{\_\_\_\_\_\_\_\_\_\_ \text{ °C} - \_\_\_\_\_\_\_\_\_\_ \text{ °C}}{\ln \left( \dfrac{\_\_\_\_\_\_\_\_\_\_ \text{ °C}}{\_\_\_\_\_\_\_\_\_\_ \text{ °C}} \right)}$$

$$\Delta T_{lm,countercurrent} = \_\_\_\_\_\_\_\_\_\_ \text{ °C}$$

For countercurrent flow, we can find the heat transfer area to be given by:

$$A_{i,countercurrent} = \frac{\_\_\_\_\_\_\_\_\_\ kW\left(\frac{1000\ W}{1\ kW}\right)}{92.4\frac{W}{m^2\cdot °C}\left(\_\_\_\_\_\_\_\_\_\ °C\right)}$$

$$\boxed{A_{i,countercurrent} = \_\_\_\_\_\_\_\_\_\ m^2}$$

To determine the heat transfer area for parallel flow, we can use Figure 4.5-3 of Geankoplis to calculate the log mean temperature difference for parallel flow, which will be given by:

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

where:

$$\Delta T_{1,parallel} = T_{air,out} - T_{ethanol,out} = \_\_\_\_\_\_\_\_\ °C - \_\_\_\_\_\_\_\_\ °C$$

$$\Delta T_{1,parallel} = \_\_\_\_\_\_\_\_\_\ °C$$

$$\Delta T_{2,parallel} = T_{air,in} - T_{ethanol,in} = \_\_\_\_\_\_\_\_\ °C - \_\_\_\_\_\_\_\_\ °C$$

$$\Delta T_{2,parallel} = \_\_\_\_\_\_\_\_\_\ °C$$

$$\Delta T_{lm,parallel} = \frac{\_\_\_\_\_\_\_\_\_\ °C - \_\_\_\_\_\_\_\_\_\ °C}{\ln\left(\frac{\_\_\_\_\_\_\_\_\_\ °C}{\_\_\_\_\_\_\_\_\_\ °C}\right)} = \_\_\_\_\_\_\_\_\_\ °C$$

Substituting the log mean temperature difference into the equation for $A_{i,parallel}$ we have:

$$A_{i,parallel} = \frac{\_\_\_\_\_\_\_\_\_\ kW\left(\frac{1000\ W}{1\ kW}\right)}{92.4\frac{W}{m^2\cdot °C}\left(\_\_\_\_\_\_\_\_\_\ °C\right)}$$

$$\boxed{A_{i,parallel} = \_\_\_\_\_\_\_\_\_\ m^2}$$

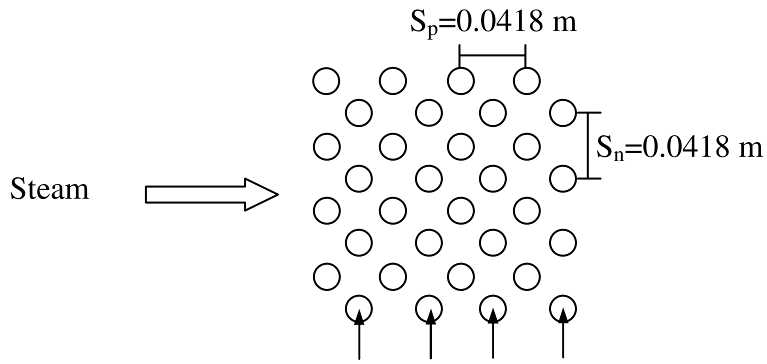
***Conclusion:***

### Example 4.6-3: Heating of Steam by a Bank of Tubes in High-Temperature Electrolysis

High-temperature electrolysis is a process for producing hydrogen from water for use in fuel cells. Before entering the electrolysis stack, steam at a pressure of 50 bar is being heated from 650°C to 850°C by a bank of 1" (nominal diameter) commercial steel tubes containing 12 rows normal to the flow and 7 staggered rows in the direction of flow. The length f the tubes is 0.4 m.

The heating medium circulating in the tubes is helium coming from a nuclear source and the outer surface of the tubes is at a temperature of 1000°C. Determine the heat-transfer rate to the steam if the velocity of steam is $16.7\frac{\text{m}}{\text{s}}$.

A diagram of this heating process is shown below:




Helium from nuclear source flowing inside the tubes

### Strategy

To solve this problem we need to calculate the amount of heat transferred by convection. The heat transfer area will depend on the number of tubes.

### Solution

The amount of heat gained by the steam can be calculated with the following equation:

$$\dot{q} = hA\left(T_w - T_b\right)$$

The bulk temperature of steam $T_b$ is obtained taking the average of the inlet and outlet temperatures:

$$T_b = \frac{850°C + 650°C}{2} = \underline{\qquad\qquad} °C$$

The temperature of the outer surface of the tube is constant and equal to 1000°C.

To calculate the amount of heat transferred, we need to obtain the heat transfer area of the tubes. The area of a single tube is calculated as follows:

$$A_{tube} = \pi DL$$

Substituting the diameter from Appendix A.5 of Geankoplis and the length of 0.4 m into this equation, we can determine the area of a single tube to be:

$$A_{tube} = \pi(\_\_\_\_\_\_\_\_\_ \text{ m})(\_\_\_\_\_\_ \text{ m})$$

$$A_{tube} = \_\_\_\_\_\_\_\_ \text{ m}^2$$

Since there are 7 columns and 12 rows of tubes, this area must be multiplied by the total number of tubes in the bank. Thus,

$$A = n_{rows} n_{columns} A_{tube} = (7)(12)(\_\_\_\_\_\_\_\_\_ \text{ m}^2)$$

$$A = \_\_\_\_\_\_\_\_\_ \text{ m}^2$$

Now we need to determine the heat transfer coefficient of steam using the correlation for flow past a bank of tubes, shown in Section 4.6 of Geankoplis.

$$h_{calculated} = \frac{k}{D} C N_{Re}{}^{m} N_{Pr}{}^{1/3}$$

In this equation, the parameters C and m will depend on the ratio of the distance between the tubes and their outer diameter. For this electrolysis process, the ratio is given by:

$$\frac{S_n}{D} = \frac{S_p}{D} = \frac{\_\_\_\_\_\_\_\_\_ \text{ m}}{\_\_\_\_\_\_\_\_\_ \text{ m}} = 1.251$$

Using Table 4.6-2 of Geankoplis, we can find the values of C and m for staggered tubes to be:

C = _________

m = _________

The dimensionless parameters $N_{Re}$ and $N_{Pr}$ are calculated from the properties of steam at the film temperature, obtained as follows:

$$T_f = \frac{T_w + T_b}{2} = \frac{\_\_\_\_\_\_\_\_\_ °C + 750°C}{2}$$

$$T_f = \_\_\_\_\_\_\_\_\_ °C$$

The properties of steam were obtained from Table 2-305 of *Perry's Chemical Engineers' Handbook*, 8th Edition and are shown below:

$$k = 0.1203 \frac{W}{m \cdot K}$$

$$C_p = 2441.3 \frac{J}{kg \cdot K}$$

$$\mu = 4.34 \times 10^{-5} \text{ Pa} \cdot \text{s}$$

$$\rho = 9.508 \frac{kg}{m^3}$$

Substituting these properties into the definition of Prandtl number we get:

$$N_{Pr} = \frac{\_\_\_\_\_\_\_\_\_\_\_\_ (\_\_\_\_\_\_\_\_\_\_ \text{ Pa} \cdot \text{s})}{\_\_\_\_\_\_\_\_\_\_\_\_}$$

$$N_{Pr} = \_\_\_\_\_\_\_\_$$

The maximum velocity required to calculate Reynolds number is obtained using the outer diameter of the pipes and the distance between the pipes normal to the direction of flow as shown in the following equation:

$$\upsilon_{max} = \frac{\upsilon S_n}{S_n - D}$$

We can enter the velocity of steam, the diameter and the distance between the pipes into this equation to yield:

$$\upsilon_{max} = \frac{16.7 \frac{m}{s} (\_\_\_\_\_\_\_\_ \text{ m})}{\_\_\_\_\_\_\_\_ \text{ m} - \_\_\_\_\_\_\_\_ \text{ m}}$$

$$\upsilon_{max} = \_\_\_\_\_\_\_\_ \frac{m}{s}$$

Now the Reynolds number can be determined as shown in the next steps:

$$N_{Re} = \frac{D \upsilon_{max} \rho}{\mu}$$

$$N_{Re} = \frac{\_\_\_\_\_\_\_\_\_ \text{ m}\left(\_\_\_\_\_\_ \frac{\text{m}}{\text{s}}\right)\left(9.508\frac{\text{kg}}{\text{m}^3}\right)}{4.34\times10^{-5}\ \text{Pa}\cdot\text{s}}$$

$$N_{Re} = \_\_\_\_\_\_\_\_\_\_\_\_$$

Substituting the dimensionless parameters we calculated and the corresponding values into the equation for $h_{calculated}$, we have:

$$h_{calculated} = \frac{k}{D} C N_{Re}{}^{m} N_{Pr}{}^{1/3}$$

$$h_{calculated} = \frac{0.1203\frac{\text{W}}{\text{m}\cdot\text{K}}}{\_\_\_\_\_\_\_\_\_\_ \text{ m}}(\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_\_\_\_\_\_)^{0.556}(\_\_\_\_\_\_\_\_)^{1/3}$$

$$h_{calculated} = \_\_\_\_\_\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}^2\cdot\text{K}}$$

The value of the heat transfer coefficient that has to be entered into the equation for the heat transfer rate has to be multiplied by a factor that depends on the amount of rows in the direction of flow.

For 7 rows and staggered tubes we can find this factor in Table 4.6-3 of Geankoplis to be _______. Thus, the actual value of the heat transferred coefficient will be given by:

$$h = \_\_\_\_\_\_ h_{calculated} = \_\_\_\_\_\_\left(\_\_\_\_\_\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)$$

$$h = \_\_\_\_\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}^2\cdot\text{K}}$$

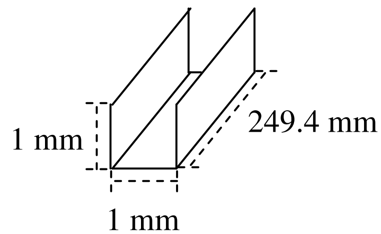
Entering this value into the equation for $\dot{q}$, we get:

$$\dot{q} = \_\_\_\_\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(\_\_\_\_\_\_\_\_\_\_\_ \text{ m}^2\right)(1000°\text{C} - 750°\text{C})$$

$$\boxed{\dot{q} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ W}}$$

### Example 4.7-3: Natural Convection in Bipolar Plate Vertical Channel

Hydrogen at standard pressure is flowing by natural convection in the bipolar plate channels of a fuel cell. These channels have a length of 24.94 cm and a thickness and height of 1 mm. Determine the heat transfer rate across the channel if the temperature of the walls is constant and equal to 82 °C. The surface of the gas diffusion layer adjacent to the channel is at a temperature of 85.17 °C. A schematic of the bipolar plate channels is shown in the following figure.




### Strategy

The heat transfer rate by convection can be calculated by using correlations that involve dimensionless groups.

### Solution

When heat is being transfer by convection, the heat transfer rate $\dot{q}$ is given by:

$$\dot{q} = hA(T_1 - T_2)$$

The heat transfer area can be calculated from the dimensions of the channel as shown below:

$$A = 2L(t + H) = 2(0.2494 \text{ m})(0.001 \text{ m} + 0.001 \text{ m})$$

$$A = \underline{\hspace{3cm}} \text{ m}^2$$

To determine the heat transfer coefficient, we can use the definition of Nusselt number:

$$N_{Nu} = \frac{hH}{k}$$

where H is the height of the channel.

However, since the Nusselt number is not given, we need to use another correlation in terms of dimensionless groups. In section 4.7 of Geankoplis, multiple correlations are shown as function of the Grashof and Prandtl numbers. We need to select the adequate correlation depending on the value yielded by the product of these dimensionless groups defined as follows:

$$N_{Gr} = \frac{H^3 \rho g \beta (T_1 - T_2)}{\mu^2}$$

In this equation:

H = height of the bipolar plate channel, m

$\rho$ = density of hydrogen, $\frac{\text{kg}}{\text{m}^3}$

g = acceleration due to gravity = $9.80665\frac{\text{m}}{\text{s}^2}$

$\beta$ = volumetric coefficient of expansion of hydrogen = $\frac{1}{T_f}$, $K^{-1}$

$\mu$ = viscosity of hydrogen, $\frac{\text{kg}}{\text{m}\cdot\text{s}}$

We can find the properties of hydrogen in Appendix A.3 of Geankoplis at the film temperature given by:

$$T_f = \frac{T_1 + T_2}{2} = \frac{\underline{\qquad\quad}\ ^\circ\text{C} + \underline{\qquad\quad}\ ^\circ\text{C}}{2}$$

$$T_f = \underline{\qquad\quad}\ ^\circ\text{C} = \underline{\qquad\qquad}\ \text{K}$$

The properties of hydrogen at this temperature (shown below) can be substituted into the equation for Grashof number to get:

$$\rho = 0.068\frac{\text{kg}}{\text{m}^3}$$

$$\mu = 9.92\times10^{-6}\frac{\text{kg}}{\text{m}\cdot\text{s}}$$

$$\beta = \frac{1}{356.74\ \text{K}} = 2.80\times10^{-3}\,\text{K}^{-1}$$

$$N_{Gr} = \frac{(0.001\,\text{m})^3\left(0.068\frac{\text{kg}}{\text{m}^3}\right)\left(\underline{\qquad\qquad}\frac{\text{m}}{\text{s}^2}\right)\left(\underline{\qquad\qquad}\ \text{K}^{-1}\right)\left(\underline{\qquad\quad}\ ^\circ\text{C} - \underline{\qquad\quad}\ ^\circ\text{C}\right)}{\left(9.92\times10^{-6}\frac{\text{kg}}{\text{m}\cdot\text{s}}\right)^2}$$

$$N_{Gr} = \underline{\qquad\quad}$$

In Appendix A.3 we can also find the value of Prandtl number for hydrogen to be:

$$N_{Pr} = \_\_\_\_\_\_\_\_$$

The product of Grashof and Prandtl numbers can now be obtained as follows:

$$N_{Pr}N_{Gr} = \_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_) = \_\_\_\_\_\_\_\_$$

Looking at the correlations in Section 4.7 of Geankoplis, we find that the Nusselt number corresponding to this value of $N_{Pr}N_{Gr}$ is 1. Thus, we can solve for the heat transfer coefficient from the definition of Nusselt number to yield:

$$h = \frac{N_{Nu}k}{H} = \frac{\_\_\_\_\_\_\_\_\left(\_\_\_\_\_\_\_\_ \frac{W}{m \cdot K}\right)}{0.001\ m}$$

$$h = \_\_\_\_\_\_\_\_ \frac{W}{m^2 \cdot K}$$

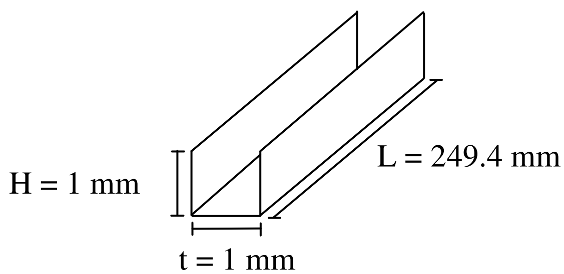
Now we can enter the corresponding quantities into the equation for the heat transfer rate to obtain:

$$\dot{q} = \_\_\_\_\_\_\_\_ \frac{W}{m^2 \cdot K}\left(\_\_\_\_\_\_\_\_\_\_\_\_\_\_ m^2\right)\left(\_\_\_\_\_\_\_\_ °C - \_\_\_\_\_\_\_\_ °C\right)$$

$$\boxed{\dot{q} = \_\_\_\_\_\_\_\_ W}$$

### Example 4.8-2: Condensation in Bipolar Plate Channels in Fuel Cells

The reaction occurring in a proton-exchange membrane fuel cell is producing water at a rate of $7.65\times10^{-5}\,\dfrac{\text{kg}}{\text{s}}$ through each channel on the bipolar plate in the cathode side. This amount of water is produced as steam at a temperature of 77°C. Determine if the water is condensing in a single channel if the partial pressure of water is 37.91 kPa. The dimensions of the channel are shown in the following figure:




The Nusselt number for a square tube with constant temperature at the boundaries is 2.98. Frano Barbir in Section 6.5.2 of the book *PEM Fuel Cells - Theory and Practice* published by Prentice Hall estimates the average temperature in the bipolar plate channels to be 64.1°C.

### Strategy

In this problem, condensation will occur if the amount of heat removed by convection is higher than the latent heat of condensation of steam.

### Solution

The amount of heat lost by the steam can be calculated from the equation for convective heat transfer as shown below:

$$\dot{q}_{conv} = hA\Delta T$$

where the change in temperature $\Delta T$ is given by:

$$\Delta T = T - T_w$$

in this equation:

T = Temperature of steam

$T_w$ = Temperature of the walls of the square channel

The convective heat transfer coefficient is obtained from the definition of Nusselt number:

$$N_{Nu} = \frac{hL}{k}$$

Solving for the convective heat transfer coefficient h and substituting the values of the length L and the thermal conductivity of steam we get:

$$h = \frac{kN_{Nu}}{L} = \frac{\_\_\_\_\_\_\_\_ \frac{W}{m \cdot K}(2.98)}{\_\_\_\_\_\_\_\_ \ m}$$

$$h = \_\_\_\_\_\_\_\_ \frac{W}{m^2 \cdot K}$$

The thermal conductivity value used in this equation was obtained from Table 2-305 of *Perry's Chemical Engineers' Handbook*, 8th Edition.

Substituting this value and the heat transfer area into the equation for $\dot{q}_{conv}$ yields:

$$A = 4(\_\_\_\_\_\_\_\_ \ m)(\_\_\_\_\_\_\_\_ \ m) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \ m^2$$

$$\dot{q}_{conv} = \_\_\_\_\_\_\_\_ \frac{W}{m^2 \cdot K}(\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \ m^2)(77°C - 64.1°C)$$

$$\dot{q}_{conv} = \_\_\_\_\_\_\_\_\_\_\_\_\_ \ W$$

To determine if the steam is condensing in the fuel cell, we need to compare this amount of heat to the latent heat of vaporization, defined as follows:

$$\dot{q}_{vap} = \dot{m}h_{fg}$$

The heat of vaporization $h_{fg}$ can be obtained from Table A.2-9 of Geankoplis. When condensation is occurring, the saturated vapor pressure is equal to the partial pressure. Hence, we will look for the enthalpy of vaporization at a pressure of 37.91 kPa and substitute into the equation for $\dot{q}_{vap}$.

$$\dot{q}_{vap} = 7.65 \times 10^{-5} \frac{kg}{s}\left(\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{J}{kg} - \_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{J}{kg}\right)$$

$$\dot{q}_{vap} = \_\_\_\_\_\_\_ \ W$$

***Conclusion:***

### Example 4.9-1: Temperature Correction Factor for a Heat Exchanger

The synthesis gas produced in a steam-methane reforming process for hydrogen production is being cooled in a heat exchanger before entering the water-gas shift reaction chamber at 846°C to 600°C. The cooling medium is air entering a heat exchanger at 255.3°C and leaving at 381.3°C. The syngas is flowing at a rate of $1322\frac{\text{kg}}{\text{hr}}$ and has a heat capacity of $2584\frac{\text{J}}{\text{kg}\cdot\text{K}}$. The air flow is being distributed through 100 commercial steel pipes with a nominal diameter of 1" Schedule number 40 and a length of 1.2 m. Calculate the mean temperature difference in the exchanger and the overall heat transfer coefficent $U_o$ for the 4 heat exchanger configurations shown in Section 4.9B of Geankoplis.

### Strategy

The heat transfer rate can be obtained using the definition of sensible heat. This can be used to determine the overall heat transfer coefficient $U_o$. The mean temperature difference will depend on the type heat exchanger selected.

### Solution

The heat transfer coefficient can be obtained from the equation shown below:

$$\dot{q} = U_o A_o \Delta T_m$$

Solving for $U_o$, we get:

$$U_o = \underline{\hspace{3cm}}$$

where:

$A_o$ = total outer surface area of the pipes distributing the air flow, $\text{m}^2$

$\Delta T_m$ = mean temperature difference = $F_T \Delta T_{lm}$, K or °C

The mean temperature difference can be obtained by multiplying a factor $F_T$ depending on the heat exchanger type by the log mean temperature difference, defined as:

$$\Delta T_{lm} = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln\frac{(T_{hi} - T_{co})}{(T_{ho} - T_{ci})}}$$

where:

$T_{hi}$ = temperature of the synthesis gas entering the heat exchanger, K or °C

$T_{ho}$ = temperature of the synthesis gas exiting the heat exchanger, K or °C

$T_{ci}$ = temperature of the air entering the heat exchanger, K or °C

$T_{co}$ = temperature of the air exiting the heat exchanger, K or °C

The amount of heat removed from the synthesis gas can be calculated as follows:

$$\dot{q} = \dot{m}C_p\left(T_{hi} - T_{ho}\right)$$

Substituting the syngas flow rate, specific heat and the inlet and outlet temperatures yields:

$$\dot{q} = 1322\frac{\text{kg}}{\text{hr}}\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right)\left(\underline{\qquad\quad}\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(\underline{\qquad}\ ^\circ\text{C} - \underline{\qquad}\ ^\circ\text{C}\right)$$

$$\dot{q} = \underline{\qquad\qquad}\ \text{W}$$

We can now determine the log mean temperature difference as shown in the following steps:

$$\Delta T_{lm} = \frac{\left(\underline{\qquad}\ ^\circ\text{C} - \underline{\qquad}\ ^\circ\text{C}\right) - \left(\underline{\qquad}\ ^\circ\text{C} - \underline{\qquad}\ ^\circ\text{C}\right)}{\ln\dfrac{\left(\underline{\qquad}\ ^\circ\text{C} - \underline{\qquad}\ ^\circ\text{C}\right)}{\left(\underline{\qquad}\ ^\circ\text{C} - \underline{\qquad}\ ^\circ\text{C}\right)}}$$

$$\Delta T_{lm} = \underline{\qquad\quad}\ ^\circ\text{C}$$

To calculate the heat transfer area of the tubes, we need to look for the outer diameter of 1" commercial steel pipes in Appendix A.5 of Geankoplis. Hence, the area can be determined as shown below:

$$A_o = \pi DLn = \pi\left(\underline{\qquad\quad}\ \text{m}\right)\left(\underline{\qquad}\ \text{m}\right)\left(\underline{\qquad}\right) = \underline{\qquad\quad}\ \text{m}^2$$

In this equation, n is the number of tubes.

Now we will proceed to calculate the mean temperature difference for the 4 different types of heat exchanger. For all heat exchanger configurations we need to calculate the parameters Y and Z, defined as:

$$Y = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}} = \frac{\underline{\qquad}\ ^\circ\text{C} - 255.3^\circ\text{C}}{\underline{\qquad}\ ^\circ\text{C} - 255.3^\circ\text{C}} \qquad Z = \frac{T_{hi} - T_{ho}}{T_{co} - T_{ci}} = \frac{846^\circ\text{C} - \underline{\qquad}\ ^\circ\text{C}}{\underline{\qquad}\ ^\circ\text{C} - 255.3^\circ\text{C}}$$

$$Y = \underline{\qquad} \qquad Z = \underline{\qquad}$$

For these values of Y and Z, we find the factors $F_T$ in Figures 4.9-4 and 4.9-5 and use them to determine the mean temperature differences and overall heat transfer coefficients, given by:

*1-2 exchanger*

$F_T =$ ______

$$\Delta T_m = F_T \Delta T_{lm} = \_\_\_\_\_\_ (\_\_\_\_\_\_\_ \text{ K})$$

$$\boxed{\Delta T_m = 391.68 \text{ K}}$$

$$U_o = \frac{\dot{q}}{A_o \Delta T_m} = \frac{\_\_\_\_\_\_\_\_\_ \text{ W}}{\_\_\_\_\_\_\_\_ \text{ m}^2 (391.68 \text{ K})}$$

$$\boxed{U_o = \_\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}^2 \cdot \text{K}}}$$

*2-4 exchanger*

$F_T =$ ___

$$\Delta T_m = F_T \Delta T_{lm} = \_\_\_\_ (\_\_\_\_\_\_\_\_\_ \text{ K})$$

$$\boxed{\Delta T_m = \_\_\_\_\_\_\_\_\_ \text{ K}}$$

$$U_o = \frac{\dot{q}}{A_o \Delta T_m} = \frac{\_\_\_\_\_\_\_\_\_ \text{ W}}{12.59 \text{m}^2 (\_\_\_\_\_\_\_\_ \text{ K})}$$

$$\boxed{U_o = \_\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}^2 \cdot \text{K}}}$$

*Cross-flow exchanger with shell fluid mixed*

$F_T =$ ______

$$\Delta T_m = F_T \Delta T_{lm} = \_\_\_\_\_ (\_\_\_\_\_\_\_\_\_ \text{ K})$$

$$\boxed{\Delta T_m = \_\_\_\_\_\_\_ \text{ K}}$$

$$U_o = \frac{\dot{q}}{A_o \Delta T_m} = \frac{\_\_\_\_\_\_\_\_\_ \text{ W}}{\_\_\_\_\_\_ \text{ m}^2 (\_\_\_\_\_\_\_\_\_ \text{ K})}$$

$$\boxed{U_o = \_\_\_\_\_\_\_\_ \frac{\text{W}}{\text{m}^2 \cdot \text{K}}}$$

*Cross-flow exchanger with fluids unmixed*

$F_T =$ _____

$$\Delta T_m = F_T \Delta T_{lm} = \_\_\_\_\_\_ (\_\_\_\_\_\_\_\_\_ \text{ K})$$

$$\boxed{\Delta T_m = \_\_\_\_\_\_\_\_\_ \text{ K}}$$

$$U_o = \frac{\dot{q}}{A_o \Delta T_m} = \frac{\_\_\_\_\_\_\_\_\_ \text{ W}}{\_\_\_\_\_\_\_\_\_ \text{ m}^2 (\_\_\_\_\_\_\_\_\_ \text{ K})}$$

$$\boxed{U_o = 47.10 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}}$$

## Example 4.9-2: Temperature Correction Factor for a Heat Exchanger

Synthesis gas is being produced in a steam-methane reforming process at a rate of $1322\frac{\text{kg}}{\text{hr}}$ and a temperature of 846°C. This gas is to be cooled to a temperature of 473.86°C before entering a water-gas shift reactor to produce additional hydrogen for fuel cells. The cooling medium is air entering at 255.3°C, a flow rate of $6307\frac{\text{kg}}{\text{hr}}$ and a heat capacity of $1058\frac{\text{J}}{\text{kg}\cdot\text{K}}$. The overall heat transfer coefficient is $90\frac{\text{W}}{\text{m}^2\cdot\text{K}}$ for a heat transfer area of 8.92 m$^2$. Determine the type of flow at which the heat exchanger is operating and the heat transfer rate if the effectiveness is 0.45. The composition of the syngas is shown in the following table:

| | wt. % |
|---|---|
| CO | 20.61 |
| $H_2$ | 7.18 |
| $H_2O$ | 72.21 |

### Strategy

The charts showing the effectiveness of heat exchangers operating at countercurrent flow and parallel flow can be used to determine the type of operation.

### Solution

The heat transfer rate in a heat exchanger can be calculated as a function of the effectiveness ε as shown in the following equation:

$$\dot{q} = \varepsilon C_{\min}\left(T_{Hi} - T_{Ci}\right)$$

To determine the value of $C_{min}$, we need to calculate the values of $C_H$ and $C_C$. These parameters depend on the flow rate and the heat capacities of the fluids in the heat exchanger. Thus,

$$C_H = \dot{m}_{\text{syngas}} C_{\text{p,syngas}}$$

$$C_C = \dot{m}_{\text{air}} C_{\text{p,air}}$$

We can see that the heat capacity of syngas is not given in the problem statement. However, we can use Figure A.3-3 of Geankoplis to determine the heat capacity of each individual component. The heat capacity of the syngas can then be determined by multiplying the mass fraction of each component by its corresponding heat capacity at the film temperature given by:

$$T_f = \frac{T_{Hi} + T_{Ho}}{2} = \frac{846°C + 473.86°C}{2}$$

$$T_f = \_\_\_\_\_\_\_\_ °C$$

Thus, at this temperature value:

$$C_{p,CO} = \_\_\_\_\_\_\_\_\_\_ \frac{J}{kg \cdot K}$$

$$C_{p,H_2} = 14853.2 \frac{J}{kg \cdot K}$$

$$C_{p,H_2O} = \_\_\_\_\_\_\_\_\_\_ \frac{J}{kg \cdot K}$$

With these individual heat capacities we can determine the heat capacity of syngas as follows:

$$C_{p,syngas} = x_{CO}C_{p,CO} + x_{H_2}C_{p,H_2} + x_{H_2O}C_{p,H_2O}$$

$$C_{p,syngas} = 0.2061\left(\_\_\_\_\_\_\_\_\_ \frac{J}{kg \cdot K}\right) + \_\_\_\_\_\_\_\_\_ \left(14853.2 \frac{J}{kg \cdot K}\right) + 0.7221\left(\_\_\_\_\_\_\_\_\_ \frac{J}{kg \cdot K}\right)$$

$$C_{p,syngas} = \_\_\_\_\_\_\_\_\_\_ \frac{J}{kg \cdot K}$$

Now we can substitute the heat capacities and flow rates of air and syngas to obtain $C_H$ and $C_C$. Thus,

$$C_H = 1322 \frac{kg}{hr}\left(\frac{1\ hr}{3600\ s}\right)\left(\_\_\_\_\_\_\_\_\_\_ \frac{J}{kg \cdot K}\right)$$

$$C_H = \_\_\_\_\_\_\_\_\_\_ \frac{W}{K}$$

$$C_C = 6307 \frac{kg}{hr}\left(\frac{1\ hr}{3600\ s}\right)\left(\_\_\_\_\_\_\_\_\_\_ \frac{J}{kg \cdot K}\right)$$

$$C_C = \_\_\_\_\_\_\_\_\_\_ \frac{W}{K}$$

We can see from the values of $C_C$ and $C_H$ that the smaller of these values is $C_H$, thus $C_{min} = C_H$. Substituting this value into the equation for the heat transfer rate, we get:

$$\dot{q} = \_\_\_\_\_\_\_\_\left(\_\_\_\_\_\_\_\_ \frac{W}{K}\right)(846°C - \_\_\_\_\_\_\_\_ °C)$$

$$\boxed{\dot{q} = \_\_\_\_\_\_\_\_\_\ kW}$$

Figure 4.9-7 of Geankoplis is showing the effectiveness of a heat exchanger operating at both countercurrent flow and parallel flow as function of the number of transfer units and the ratio $\frac{C_{min}}{C_{max}}$.

If we calculate these two values, we can look in this Figure which type of heat exchanger will yield a value of $\varepsilon = \_\_\_\_\_\_\_\_$ . Thus,

$$\frac{C_{min}}{C_{max}} = \frac{\_\_\_\_\_\_\_\_ \frac{W}{K}}{\_\_\_\_\_\_\_\_ \frac{W}{K}} = 0.59$$

$$NTU = \frac{UA}{C_{min}} = \frac{90 \frac{W}{m^2 \cdot K}(8.92 m^2)}{\_\_\_\_\_\_\_\_ \frac{W}{K}}$$

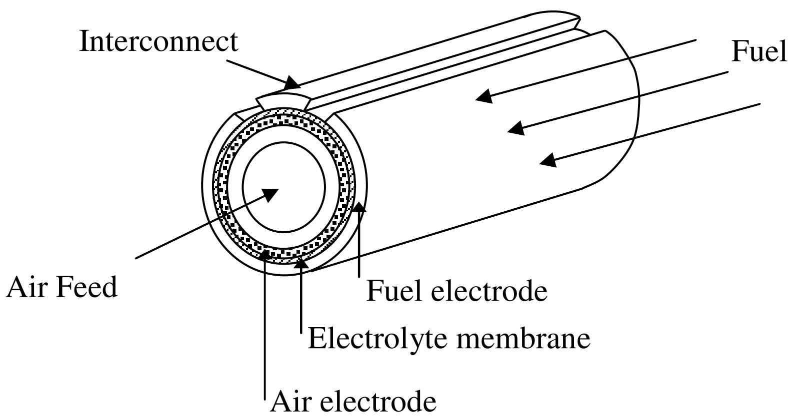
$$NTU = \_\_\_\_\_\_\_\_$$

***Conclusion:***

### Example 4.11-1: Radiation in Cylindrical Solid - Oxide Fuel Cell

The following figure is a schematic of a cylindrical solid-oxide fuel cell. Jiang et al. [1] developed a thermoelectrical model to estimate the temperature at different parts of this type of fuel cell. The temperatures are estimated to be 1125 K for the air tube and 1200 K for the solid part (membrane electrode assembly). Determine the heat flux due to radiation.

Xue et al. [2] estimated the average emissivity of the membrane electrode assembly to be 0.33. The air is being fed to the system through a commercial steel pipe.




### Strategy

The heat transferred due to radiation can be estimated using the radiation equation for gray bodies given in Section 4.11 of Geankoplis.

### Solution

The following equation is the definition of heat flux due to radiation:

$$\dot{q}'' = \sigma\left(T_1^4 - T_2^4\right)\left(\frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}\right)$$

where:

$\dot{q}''$ = Heat flux due to radiation, $\frac{W}{m^2}$

1. Jiang, W., Fang, R., Dougal, R. A., Khan, J. A., *Journal of Energy Resources Technology*, 130, 2008.

2. Xue, X., Tang, J., Sammes, N., Du, Y., *Journal of Power Sources*, 142, 211−222 (2005)

$$\sigma = \text{Stefan} - \text{Boltzmann constant} = 5.676\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}$$

$T_i$ = Temperature of surface *i*, K

$\varepsilon_i$ = Emissivity of surface *i*, K

We can substitute the given quantities into the equation for heat flux to solve this problem. Hence,

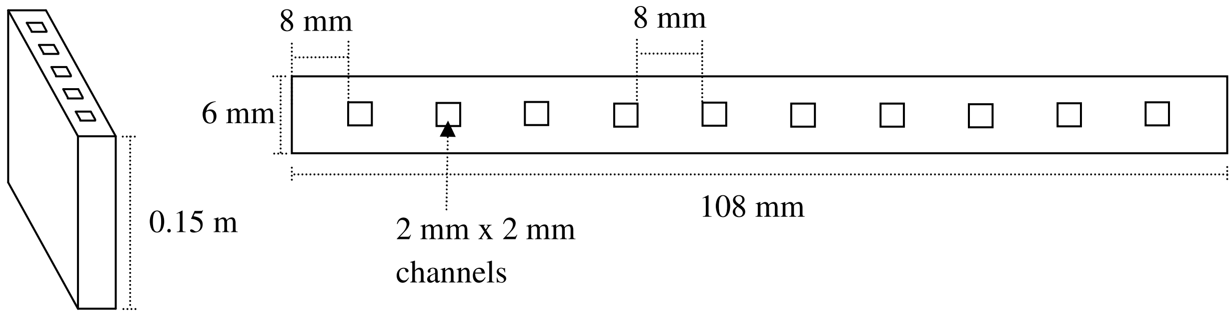
$$\dot{q}'' = \underline{\hspace{4cm}}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left[\left(\underline{\hspace{1.5cm}}\ \text{K}\right)^4 - \left(1125\ \text{K}\right)^4\right]\left(\frac{1}{1/\underline{\hspace{1.2cm}} + 1/\underline{\hspace{1.2cm}} - 1}\right)$$

$$\boxed{\dot{q}'' = \underline{\hspace{3cm}}\ \frac{\text{W}}{\text{m}^2}}$$

The emissivity of steel was obtained from Table 5−4 of *Perry's Chemical Engineers' Handbook*, 8th Edition.

### 4.15-1 Cooling Channels in Fuel Cell Bipolar Plates

The following figures show the top and isometric views of a fuel cell bipolar plate with 10 cooling channels through which air is circulating with a heat transfer coefficient of $400\dfrac{\text{W}}{\text{m}^2\cdot\text{K}}$ and a temperature of 10°C. The outer walls of the bipolar plates are held at a temperature of 60°C. Determine the steady – state heat loss in one bipolar plate using finite difference numerical methods, with grids 1 mm x 1 mm. The bipolar plates are made of 304 stainless steel.

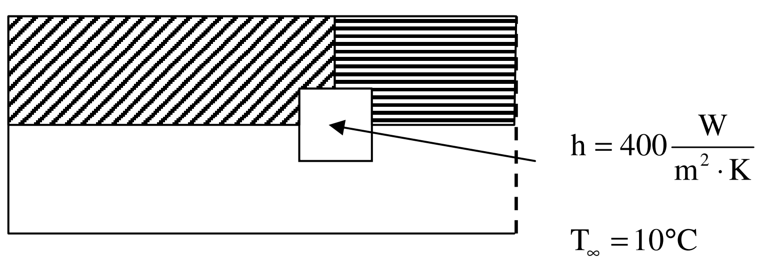



### Strategy

To determine the amount of heat removed we need to determine the temperatures at the different nodes, using the Equations in Section 4.15B of Geankoplis.

### Solution

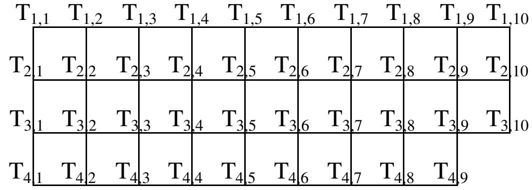
Since the area surrounding the channel is symmetrical, we can calculate it for one channel and multiply it by the number of times this area is repeated in the whole bipolar plate. If we zoom into the first channel from the left edge of the bipolar plate:




The shaded areas in this figure indicate the sets of nodes that will be repeated along the bipolar plate.

*Set of Nodes at the Edges of the Bipolar Plate*

We will start by using the finite difference method on the edges of the bipolar plate. For a grid of 1 mm by 1 mm, the following nodes will be used:



The outer nodes are kept at a temperature of 60°C. Thus:

$$T_{1,1} = T_{1,2} = T_{1,3} = T_{1,4} = T_{1,5} = T_{1,6} = T_{1,7} = T_{1,8} = T_{1,9} = T_{1,10} = T_{2,1} = T_{3,1} = T_{4,1} = \_\_\_\_\_ °C$$

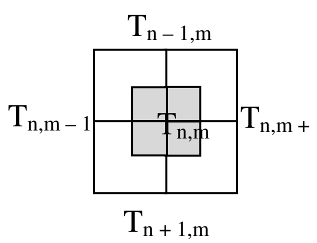
For the first calculation, we will assume the following temperature values (in °C) for the rest of the nodes:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $T_{1,1}$ | $T_{1,2}$ | $T_{1,3}$ | $T_{1,4}$ | $T_{1,5}$ | $T_{1,6}$ | $T_{1,7}$ | $T_{1,8}$ | $T_{1,9}$ | $T_{1,10}$ |
| $T_{2,1}$ | $T_{2,2}$ | $T_{2,3}$ | $T_{2,4}$ | $T_{2,5}$ | $T_{2,6}$ | $T_{2,7}$ | $T_{2,8}$ | $T_{2,9}$ | $T_{2,10}$ |
| $T_{3,1}$ | $T_{3,2}$ | $T_{3,3}$ | $T_{3,4}$ | $T_{3,5}$ | $T_{3,6}$ | $T_{3,7}$ | $T_{3,8}$ | $T_{3,9}$ | $T_{3,10}$ |
| $T_{4,1}$ | $T_{4,2}$ | $T_{4,3}$ | $T_{4,4}$ | $T_{4,5}$ | $T_{4,6}$ | $T_{4,7}$ | $T_{4,8}$ | $T_{4,9}$ | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| 60 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 35 |
| 60 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 10 |
| 60 | 45 | 40 | 35 | 30 | 25 | 20 | 15 | 10 | |

We can obtain the first temperature estimation for the interior nodes using Equation 4.15-11 from Geankoplis:

$$q_{n,m} = T_{n-1,m} + T_{n+1,m} + T_{n,m-1} + T_{n,m+1} - 4T_{n,m}$$




This equation is applicable to the nodes highlighted below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $T_{1,1}$ | $T_{1,2}$ | $T_{1,3}$ | $T_{1,4}$ | $T_{1,5}$ | $T_{1,6}$ | $T_{1,7}$ | $T_{1,8}$ | $T_{1,9}$ | $T_{1,10}$ |
| $T_{2,1}$ | $T_{2,2}$ | $T_{2,3}$ | $T_{2,4}$ | $T_{2,5}$ | $T_{2,6}$ | $T_{2,7}$ | $T_{2,8}$ | $T_{2,9}$ | $T_{2,10}$ |
| $T_{3,1}$ | $T_{3,2}$ | $T_{3,3}$ | $T_{3,4}$ | $T_{3,5}$ | $T_{3,6}$ | $T_{3,7}$ | $T_{3,8}$ | $T_{3,9}$ | $T_{3,10}$ |
| $T_{4,1}$ | $T_{4,2}$ | $T_{4,3}$ | $T_{4,4}$ | $T_{4,5}$ | $T_{4,6}$ | $T_{4,7}$ | $T_{4,8}$ | $T_{4,9}$ | |

To start the calculations, we will select node $T_{2,2}$. We can apply this equation to get:

$$q_{2,2} = T_{1,2} + \_\_\_\_\_\_ + T_{2,1} + \_\_\_\_\_\_ - 4T_{2,2}$$

$$q_{2,2} = \_\_\_\_ + 50 + \_\_\_\_ + 55 - 4(\_\_\_\_) = \_\_\_\_$$

Since the heat $q_{2,2}$ is not equal to zero, the value of $T_{2,2}$ we assumed is not the temperature at steady state. Setting the value of $q_{2,2}$ to zero we calculate a new value of $T_{2,2}$ as shown below:

$$T_{2,2} = \frac{T_{1,2} + \underline{\qquad} + T_{2,1} + \underline{\qquad}}{4}$$

$$T_{2,2} = \frac{\underline{\qquad}\,°C + 50°C + \underline{\qquad}\,°C + \underline{\qquad}\,°C}{4} = \underline{\qquad\qquad}\ °C$$

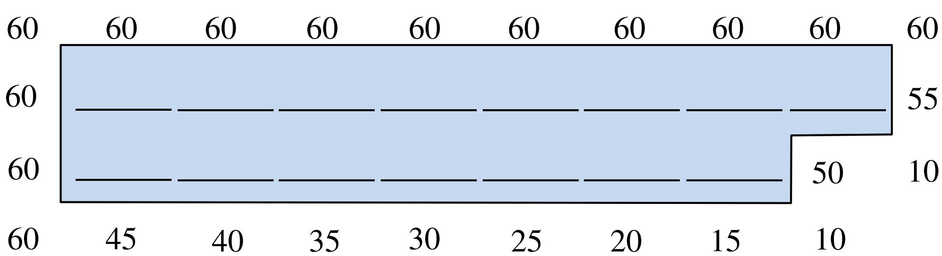
This new value of $T_{2,2}$ will be used to calculate the temperatures at other nodes. Thus, for $q_{2,3}$:

$$q_{2,3} = \underline{\qquad} + T_{3,3} + \underline{\qquad} + T_{2,4} - \underline{\qquad\quad} = 60 + 50 + \underline{\qquad\quad} + 55 - \underline{\qquad\quad} = \underline{\qquad\quad}$$

Setting $q_{3,2}$ = 0 and solving for $T_{3,2}$ yields:

$$T_{2,3} = \frac{60 + 50 + \underline{\qquad\quad} + 55}{4} = \underline{\qquad\quad}\ °C$$

We can repeat the same procedure for all the interior nodes. The first iteration will yield the following temperature values:




Note that we have not done any calculations for the edge nodes. This is because we need additional equations for these nodes, described in the following sections.

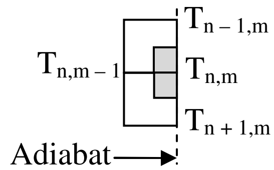
Section 4.15B-3 of Geankoplis and Section 4.5-3 of Incropera and DeWitt [3] gives the following equations for different boundary conditions.

For nodes $T_{2,10}$, $T_{4,2}$, $T_{4,3}$, $T_{4,4}$, $T_{4,5}$, $T_{4,6}$, $T_{4,7}$, and $T_{4,8}$, an equation with an adiabatic boundary is needed [3].

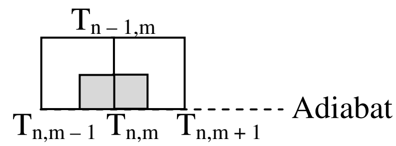
**3.** Incropera, F. P., DeWitt, D. P., Fundamentals of Heat and Mass Transfer, Fourth Edition, John Wiley & Sons, New York (1996).

This equation is applied for nodes with heat conduction from three adjacent nodes with an adjoining adiabat, as shown in the following figures:

$$q_{n,m} = 2T_{n,m-1} + T_{n-1,m} + T_{n+1,m} - 4T_{n,m}$$

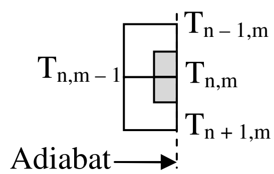



$$q_{n,m} = 2\underline{\qquad\quad} + T_{n,m-1} + T_{n,m+1} - 4\underline{\qquad\quad}$$

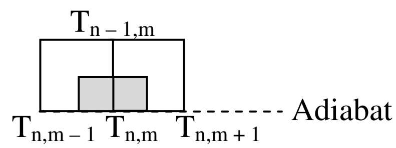



Setting $q_{n,m} = 0$ in these equations, the temperatures are given by:

$$T_{n,m} = \frac{2T_{n,m-1} + \underline{\qquad\quad} + T_{n+1,m}}{\underline{\quad}-}$$




$$T_{n,m} = \frac{2\underline{\qquad\quad} + T_{n,m-1} + T_{n,m+1}}{\underline{\qquad}}$$




Applying this equation to node $T_{4,2}$ (with $q_{4,2} = 0$), we have:

$$T_{4,2} = \frac{2\underline{\qquad} + T_{4,1} + \underline{\qquad}}{\underline{\quad}} = \frac{2(\underline{\qquad}\,^\circ C) + 60^\circ C + 40^\circ C}{\underline{\quad}} = \underline{\qquad}\,^\circ C$$

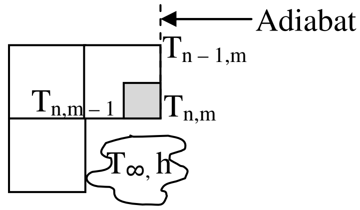
In a similar way, we can find the first estimate of the temperatures at rest of the nodes that follow this equation, highlighted in the grid below:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $T_{1,1}$ | $T_{1,2}$ | $T_{1,3}$ | $T_{1,4}$ | $T_{1,5}$ | $T_{1,6}$ | $T_{1,7}$ | $T_{1,8}$ | $T_{1,9}$ | $T_{1,10}$ |
| $T_{2,1}$ | $T_{2,2}$ | $T_{2,3}$ | $T_{2,4}$ | $T_{2,5}$ | $T_{2,6}$ | $T_{2,7}$ | $T_{2,8}$ | $T_{2,9}$ | $T_{2,10}$ |
| $T_{3,1}$ | $T_{3,2}$ | $T_{3,3}$ | $T_{3,4}$ | $T_{3,5}$ | $T_{3,6}$ | $T_{3,7}$ | $T_{3,8}$ | $T_{3,9}$ | $T_{3,10}$ |
| $T_{4,1}$ | $T_{4,2}$ | $T_{4,3}$ | $T_{4,4}$ | $T_{4,5}$ | $T_{4,6}$ | $T_{4,7}$ | $T_{4,8}$ | $T_{4,9}$ | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| 60 | ______ | ______ | ______ | ______ | ______ | ______ | ______ | ______ | ______ |
| 60 | ______ | ______ | ______ | ______ | ______ | ______ | ______ | 50 | 10 |
| 60 | ______ | ______ | ______ | ______ | ______ | ______ | ______ | 10 | |

To finish the first temperature estimation, we need to determine 3 more nodes: $T_{3,9}$, $T_{3,10}$ and $T_{4,9}$. In nodes $T_{4,9}$ and $T_{3,10}$, heat transfer is occurring at an insulated boundary with convection from internal flow and conduction from the two adjacent nodes. The following equation (at steady state) can be used for this case, also given in page 191 of Incropera [3]. The thermal conductivity of stainless steel 304 was obtained from Appendix A.3 of Geankoplis.

$$T_{n,m} = \frac{T_{n,m-1} + T_{n-1,m} + \left(\frac{h\Delta x}{k}\right) T_\infty}{\left(2 + \frac{h\Delta x}{k}\right)}$$




Substituting the corresponding temperatures into this equation yields:

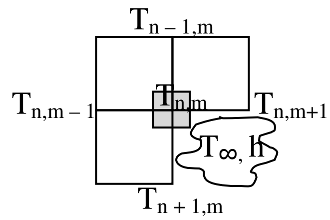
$$T_{3,10} = \frac{T_{3,9} + \underline{\qquad} + \left(\frac{h\Delta x}{k} T_\infty\right)}{\left(2 + \frac{h\Delta x}{k}\right)} = \frac{\underline{\quad}\ {}^\circ C + 45^\circ C + \left[\dfrac{400\frac{W}{m^2\cdot K}(\underline{\qquad}\ m)}{\underline{\qquad}\ \frac{W}{m\cdot K}}(10^\circ C)\right]}{\left[2 + \dfrac{400\frac{W}{m^2\cdot K}(\underline{\qquad}\ m)}{\underline{\qquad}\ \frac{W}{m\cdot K}}\right]} = \underline{\qquad}\ {}^\circ C$$

Now for $T_{4,9}$ we have:

$$T_{4,9} = \frac{T_{4,8} + T_{3,9} + \left(\frac{h\Delta x}{k} T_\infty\right)}{\left(2 + \frac{h\Delta x}{k}\right)} = \frac{31.3^\circ C + \underline{\quad}\ {}^\circ C + \left[\dfrac{400\frac{W}{m^2\cdot K}(\underline{\qquad}\ m)}{\underline{\qquad}\ \frac{W}{m\cdot K}}(10^\circ C)\right]}{\left[2 + \dfrac{400\frac{W}{m^2\cdot K}(\underline{\qquad}\ m)}{\underline{\qquad}\ \frac{W}{m\cdot K}}\right]} = \underline{\qquad}\ {}^\circ C$$

For node $T_{3,9}$, we can use Equation 4.15-19 of Geankoplis, which corresponds to an interior corner with convection at the boundary:

$$T_{n,m} = \frac{\frac{1}{2}\left(T_{n+1,m} + T_{n,m+1}\right) + T_{n,m-1} + T_{n-1,m} + \left(\frac{h\Delta x}{k} T_\infty\right)}{\left(3 + \frac{h\Delta x}{k}\right)}$$




$$T_{3,9} = \frac{\frac{1}{2}\left(T_{4,9} + \_\_\_\_\_\_\right) + T_{3,8} + \_\_\_\_\_\_ + \left(\frac{h\Delta x}{k} T_\infty\right)}{\left(3 + \frac{h\Delta x}{k}\right)}$$

$$T_{3,9} = \frac{\frac{1}{2}\left(\_\_\_\_\_\_\,°C + \_\_\_\_\_\_\,°C\right) + 40.56°C + 55°C + \left[\frac{400\frac{W}{m^2\cdot K}\left(\_\_\_\_\_\_\ m\right)}{\_\_\_\_\_\_\frac{W}{m\cdot K}}(10°C)\right]}{\left[3 + \frac{400\frac{W}{m^2\cdot K}\left(\_\_\_\_\_\_\ m\right)}{\_\_\_\_\_\_\frac{W}{m\cdot K}}\right]}$$

$$T_{3,9} = \_\_\_\_\_\_\,°C$$

The following grid shows the temperatures obtained after the first iteration. The highlighted temperatures are the values we just calculated.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| 60 | ______ | ______ | ______ | ______ | ______ | ______ | ______ | ______ | ______ |
| 60 | ______ | ______ | ______ | ______ | ______ | ______ | ______ | ______ | ______ |
| 60 | ______ | ______ | ______ | ______ | ______ | ______ | ______ | ______ | |

After completing the first calculation across the grid, we can start a new approximation using the new temperature values. Hence, starting with $q_{2,2}$ and $T_{2,2}$, we have:

$$q_{2,2} = T_{1,2} + T_{3,2} + T_{2,1} + T_{2,3} - 4T_{2,2}$$

$$q_{2,2} = 60 + 52.81 + 60 + \_\_\_\_\_\_ - 4\left(\_\_\_\_\_\_\right) = \_\_\_\_\_\_$$

Setting the value of $q_{2,2}$ to zero we calculate a new value of $T_{2,2}$ as follows:

$$T_{2,2} = \frac{60 + \_\_\_\_\_\_ + 60 + \_\_\_\_\_\_}{4} = \_\_\_\_\_\_ \text{ °C}$$

The same procedure used for the first approximation is repeated until the assumed and new temperatures are similar. In this case we will select a tolerance of 0.01.

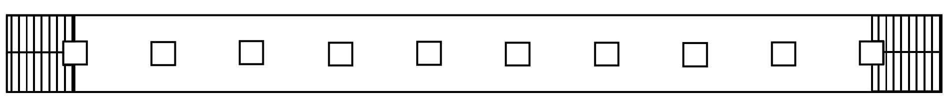
This numerical problem can also be solved using computer software such as Excel or Matlab. The final temperature values in the set of nodes for the edges of the bipolar plate are shown below:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 60 | | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| 60 | ______ | 59.98 | ______ | ______ | 59.88 | ______ | ______ | 59.55 | ______ |
| 60 | ______ | ______ | 59.93 | ______ | ______ | 59.65 | ______ | ______ | 58.88 |
| 60 | 59.98 | ______ | ______ | 59.86 | ______ | ______ | 59.30 | ______ | |

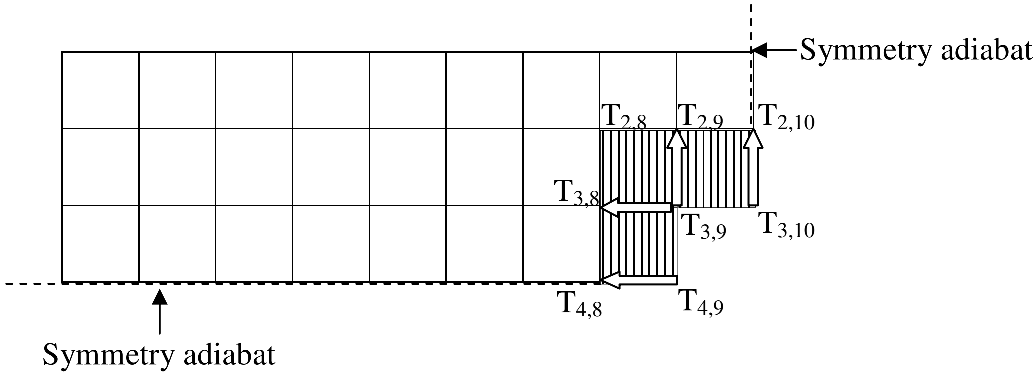
To calculate the total heat lost by the bipolar plate we use Fourier's Law of Heat Conduction for the interior and exterior nodes.

$$q = kA\frac{\Delta T}{\Delta x} = k\Delta xL\frac{\Delta T}{\Delta x} = kL(\Delta T)$$

This amount must be multiplied by 4, since this set of nodes is repeated 4 times in the fuel cell bipolar plate (4 external corners), as shown in the shaded areas below:



The total heat conducted is the sum of the heat equations applied for all the interior temperature differences. The following figure illustrates the nodes used for the heat loss calculation. The nodes adjacent to the shaded squares were used for determining $q_{interior}$, with a direction for heat flow indicated by the arrows. The heat flux for nodes with an adjoining adiabat has to be multiplied by $\frac{1}{2}$ because of symmetry.



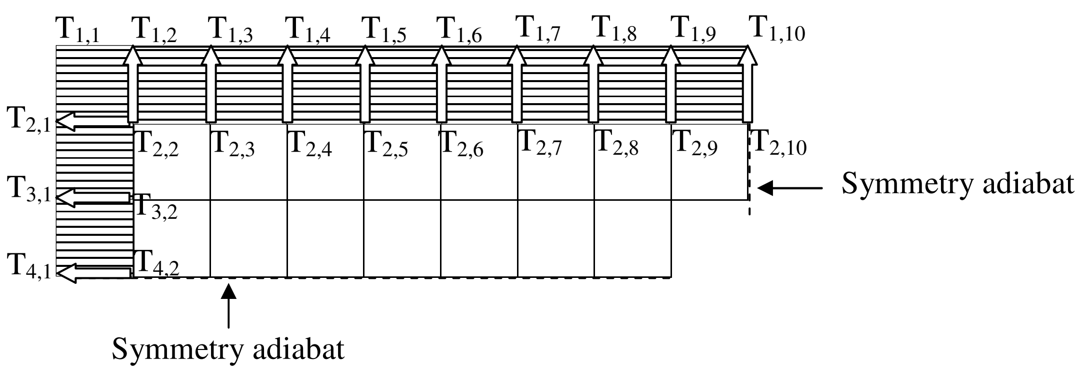

Hence, for the interior nodes we have:

$$q_{interior,corner} = 4kL\left[0.5\left(T_{3,10} - T_{2,10}\right) + 0.5\left(T_{4,9} - T_{4,8}\right) + (\_\_\_\_ - \_\_\_\_) + (\_\_\_\_ - \_\_\_\_)\right]$$

$$q_{interior,corner} = 4\left(16.3\frac{\text{W}}{\text{m}\cdot\text{K}}\right)(\_\_\_\_\ \text{m})\left[0.5(\_\_\_\_ - 59.50) + 0.5(58.78 - \_\_\_\_)\right.$$
$$\left. + (\_\_\_\_ - 59.55) + (59.02 - \_\_\_\_)\right]°\text{C}$$

$$q_{interior,corner} = \_\_\_\_\_\_\ \text{W}$$

To determine $q_{exterior}$, we substituted the temperature differences in the nodes adjacent to the shaded squares in the following figure. The direction of heat flow is indicated by the arrows , with a The heat flux for nodes with an adjoining adiabat has to be multiplied by $\frac{1}{2}$ because of symmetry.



Applying Fourier's law to the exterior nodes, the heat transfer rate will be given by:

$$\begin{aligned} q_{\text{exterior,corner}} = 4kL\Big[&0.5\left(\underline{\qquad} - T_{1,10}\right) + 0.5\left(T_{4,2} - \underline{\qquad}\right) + \left(\underline{\qquad} - T_{1,9}\right) + \left(T_{2,8} - \underline{\qquad}\right) \\ &+ \left(\underline{\qquad} - T_{1,7}\right) + \left(T_{2,6} - \underline{\qquad}\right) + \left(\underline{\qquad} - T_{1,5}\right) + \left(T_{2,4} - T_{1,4}\right) + \left(T_{2,3} - T_{1,3}\right) \\ &+ \left(T_{2,2} - T_{1,2}\right) + \left(T_{2,2} - T_{2,1}\right) + \left(T_{3,2} - T_{3,1}\right)\Big] \end{aligned}$$

$$\begin{aligned} q_{\text{exterior,corner}} = 4\left(16.3\frac{\text{W}}{\text{m}\cdot\text{K}}\right)(\underline{\qquad}\ \text{m})\Big[&0.5(\underline{\qquad} - 60) + 0.5(\underline{\qquad} - 60) + (\underline{\qquad} - 60) \\ &+ (\underline{\qquad} - 60) + (\underline{\qquad} - 60) + (\underline{\qquad} - 60) + (\underline{\qquad} - 60) + (\underline{\qquad} - 60) \\ &+ (\underline{\qquad} - 60) + (\underline{\qquad} - 60) + (\underline{\qquad} - 60) + (\underline{\qquad} - 60)\Big]^{\circ}\text{C} \end{aligned}$$

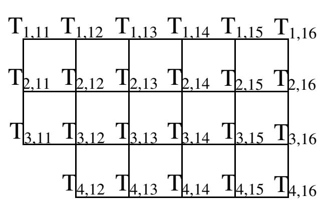
$$q_{\text{exterior,corner}} = \underline{\qquad\qquad}\ \text{W}$$

Finding the same value as $q_{\text{interior, corner}}$ proves that this system is at steady – state. The heat transfer rate for this set of nodes will be obtained from the average between the heat transfer for the interior and exterior nodes.

$$q_{\text{corners}} = \underline{\qquad\qquad}\ \text{W}$$

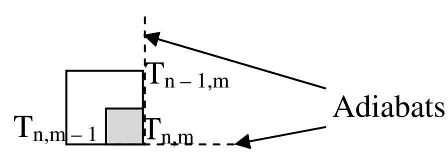
*Set of nodes between cooling channels*

We need to establish a different nodal network for the spaces between cooling channels. The set of nodes for a 1 mm x 1 mm grid is shown below:




The same equations used for calculating the temperatures at the exterior corners of the bipolar plates will be used again for this set of nodes. The only exception is for node $T_{4,16}$, which represents the case for heat conduction with two adjoining adiabats. Thus, for this node (setting $q_{4,16} = 0$) we have:

$$T_{n,m} = \frac{T_{n,m-1} + T_{n-1,m}}{2}$$

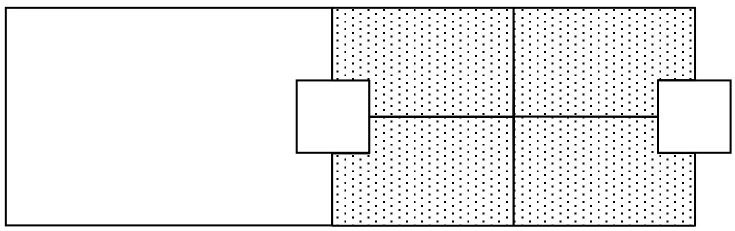



$$T_{4,16} = \frac{\underline{\qquad} + T_{4,15}}{2}$$

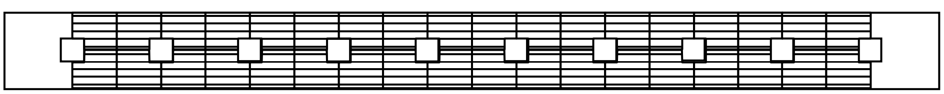
Following the same procedure described for the first set of nodes in this problem (for exterior corners), or using computer software, we can find the temperatures for the interior nodes to be:

| | | | | | | |
|---|---|---|---|---|---|---|
| $T_{1,11}$ $T_{1,12}$ $T_{1,13}$ $T_{1,14}$ $T_{1,15}$ $T_{1,16}$ | 60 | 60 | 60 | 60 | 60 | 60 |
| $T_{2,11}$ $T_{2,12}$ $T_{2,13}$ $T_{2,14}$ $T_{2,15}$ $T_{2,16}$ | 59.49 | ______ | ______ | 59.78 | ______ | ______ |
| $T_{3,11}$ $T_{3,12}$ $T_{3,13}$ $T_{3,14}$ $T_{3,15}$ $T_{3,16}$ | ______ | 58.99 | ______ | ______ | 59.71 | ______ |
| $T_{4,12}$ $T_{4,13}$ $T_{4,14}$ $T_{4,15}$ $T_{4,16}$ | | ______ | 59.26 | ______ | ______ | 59.71 |

Now we can proceed to calculate the heat transfer rate using Fourier's law for the temperature difference at the interior and exterior nodes. If we look at the top view of the bipolar plates, we can see that this set of nodes is repeated 4 times in the space between two channels as illustrated below.



Since there are 10 channels, there will be 9 spaces between the cooling channels (see figure below). Therefore the heat transfer rate in a single set of nodes must be multiplied by 36 to obtain the heat transfer in the whole bipolar plate.



In a similar way as we did for the set of nodes for the corners of the bipolar plate, we can determine the heat flow for the set of nodes including the spaces between cooling channels. Thus,

$$q_{\text{interior,middle}} = 36kL\left[0.5\left(T_{3,11}-T_{2,11}\right)+0.5\left(\underline{\hspace{1.5cm}}-\underline{\hspace{1.5cm}}\right)+\left(\underline{\hspace{1.5cm}}-\underline{\hspace{1.5cm}}\right)+\left(T_{3,12}-T_{3,13}\right)\right]$$

$$q_{\text{interior,middle}} = 36\left(16.3\frac{\text{W}}{\text{m}\cdot\text{K}}\right)\left(\underline{\hspace{1.5cm}}\ \text{m}\right)\left[0.5\left(\underline{\hspace{1.5cm}}-59.49\right)+0.5\left(\underline{\hspace{1.5cm}}-59.26\right)\right.$$
$$\left.+\left(58.99-\underline{\hspace{1.5cm}}\right)+\left(58.99-\underline{\hspace{1.5cm}}\right)\right]^{\circ}\text{C}$$

$$q_{\text{interior,middle}} = \underline{\hspace{2cm}}\ \text{W}$$

Similarly for exterior nodes, we have:

$$q_{exterior,middle} = 36\left(16.3\frac{W}{m\cdot K}\right)(\_\_\_\_\_\_\ m)\big[0.5(\_\_\_\_\_ - \_\_\_\_\_) + 0.5(\_\_\_\_\_ - \_\_\_\_\_) + (T_{2,12} - T_{1,12})$$
$$+(\_\_\_\_\_ - \_\_\_\_\_) + (T_{2,14} - T_{1,14}) + (\_\_\_\_\_ - \_\_\_\_\_) + (\_\_\_\_\_ - \_\_\_\_\_)$$
$$+(\_\_\_\_\_ - \_\_\_\_\_)\big]$$

$$q_{exterior,middle} = 36\left(16.3\frac{W}{m\cdot K}\right)(0.15\ m)\big[0.5(59.49-60) + 0.5(\_\_\_\_\_\_ - 59.71) + (\_\_\_\_\_\_ - 60)$$
$$+(\_\_\_\_\_\_ - 60) + (59.78-60) + (\_\_\_\_\_\_ - 60) + (\_\_\_\_\_\_ - \_\_\_\_\_\_)$$
$$+(59.71 - \_\_\_\_\_\_)\big]°C$$

$$q_{exterior,middle} = \_\_\_\_\_\_\_\ W$$

Taking the average of $q_{interior,middle}$ and $q_{exterior,middle}$ we get:

$$q_{middle} = \frac{\_\_\_\_\_\_\_\ W - \_\_\_\_\_\_\_\ W}{2}$$

$$q_{middle} = \_\_\_\_\_\_\_\ W$$

To obtain the overall heat transfer in the bipolar plate we need to add $q_{corners}$ and $q_{middle}$:

$$q = \_\_\_\_\_\_\_\ W - \_\_\_\_\_\_\_\ W$$

$$\boxed{q = \_\_\_\_\_\_\_\_\ W}$$