Chapter 3

Principles of Momentum Transfer and Applications

Chapter 3 introduces students to the principles for calculation of power and efficiency of equipment such as compressors, pumps and fans. The following problem modules illustrate the application of this type of equipment to processes for producing hydrogen for fuel cells as well as the derivation of equations for different flow conditions from the general transport equations.

3.1-3 Surface Area in Packed Bed of Cylinders
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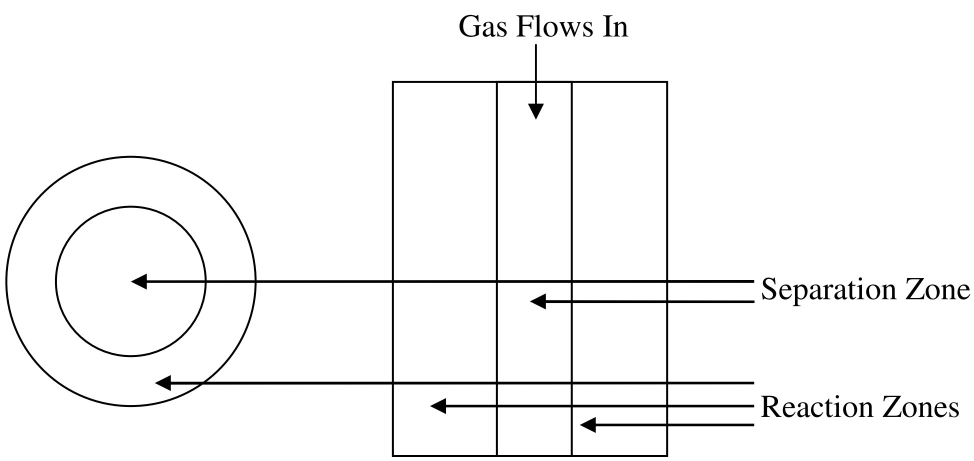
Example 3.1-3: Surface Area in Packed Bed of Cylinders

Natural gas has been proposed as a source of hydrogen for fuel cell vehicle applications because of the existing infrastructure. In a process known as steam reforming, natural gas and steam are reacted into mostly carbon monoxide and hydrogen with some carbon dioxide also produced. There is also excess water in the reformate stream.

A water gas shift reactor can be used to convert some of the remaining carbon monoxide into hydrogen according to the reaction:

$$CO + H_2O \rightleftharpoons H_2 + CO_2$$

The following figure shows an axisymmetric view of an annular water gas shift reactor which is 8 cm high. In the outer (annular) region, an iron chromium oxide catalyst is present to carry out the water gas shift reaction. A 20 μm thick palladium membrane separates the reaction (outer) zone from the separation (inner) zone.



Annular Membrane Reactor : Top View (Left) and Side View (Right)

Cylinders of iron chromium oxide catalyst with diameter and length of 0.1 cm are forming a packed bed in the reaction zones with a bulk density of $39.75\frac{\text{lb}}{\text{ft}^3}$. Determine the void fraction ε, the effective diameter of the particles D_p and the hydraulic radius r_H for the flow through the packed bed if the density of the catalyst is $76\frac{\text{lb}}{\text{ft}^3}$.

Strategy

The equations defining the parameters ε, D_p and r_H can be used for solving this problem.

Solution

First we will calculate the void fraction ε of the packed bed, defined by the following equation:

$$\varepsilon = \frac{\text{volume of voids in bed}}{\text{total volume of bed}}$$

For simplicity, we will select a basis of 1 ft^3 of packed bed. Thus, we can calculate the mass of the bed to be:

$$m_{bed} = 1\ ft^3 \left(\_\_\_\_\_\_\_\_\_\_ \frac{lb}{ft^3} \right) = \_\_\_\_\_\_\_\_\_\_\_\_\ \ lb$$

This mass of packed bed can be used to calculate the volume of the solid cylinders of catalyst, as shown in the following calculation:

$$V_{catalyst} = \frac{\_\_\_\_\_\_\_\_\_\ lb}{76 \frac{lb}{ft^3}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\ \ ft^3$$

Substituting this volume and the basis of 1 ft^3 of packed bed into the equation for ε yields:

$$\varepsilon = \frac{1\ ft^3 - \_\_\_\_\_\_\_\_\_\_\_\_\ ft^3}{\_\_\_\_\_\_\ ft^3}$$

$$\boxed{\varepsilon = \_\_\_\_\_\_\_\_\_\_}$$

For non-spherical particles, the effective diameter is given by the following equation:

$$D_p = \frac{6}{a_V}$$

where a_V is the specific area, given by the ratio of the surface area of the catalyst particle to the volume of the particle. Thus, a_V is given by:

$$a_V = \frac{S_p}{V_p} = \frac{2\left[\frac{\pi \_\_\_\_\_\_\_}{\_\_\_\_\_}\right] + \pi \_\_\_\_\_\_\_\_\_\_\_\_}{\frac{\pi D^2}{4} \_\_\_\_\_\_}$$

In this equation, the first term in the numerator represents the area of the ends of the cylinder, while the second term is accounting for the area of the walls of the cylinder. We can substitute the values of D and L into this equation to determine the specific area a_V as follows:

$$a_V = \frac{S_p}{V_p} = \frac{2\left[\frac{\pi \underline{\hspace{3cm}}}{\underline{\hspace{2cm}}}\right] + \pi(\underline{\hspace{2.5cm}})^2}{\frac{\pi(0.1\text{ cm})^2}{4}(\underline{\hspace{3cm}})} = 60\frac{1}{\text{cm}}\left(\frac{\underline{\hspace{2.5cm}}\text{ cm}}{1\text{ ft}}\right)$$

$$a_V = 1828.8\text{ ft}^{-1}$$

Now we can calculate the effective diameter to be:

$$D_p = \frac{6}{1828.8\text{ ft}^{-1}}$$

$$\boxed{D_p = \underline{\hspace{3cm}}\text{ ft}}$$

The hydraulic radius of an object is defined as:

$$r_H = \frac{\varepsilon}{a}$$

where a is the ratio of the wetted surface of the particles to the volume of the packed bed. The following equation can be used for calculation of a:

$$a = a_V(1-\varepsilon)$$

The values of the void fraction and the specific area of the catalyst particles can be entered into this equation to give:

$$a = 1828.8\text{ ft}^{-1}(1-\underline{\hspace{3cm}})$$

$$a = \underline{\hspace{3cm}}\text{ ft}^{-1}$$

Therefore, the hydraulic radius can now be calculated to yield:

$$r_H = \frac{\underline{\hspace{3cm}}}{\underline{\hspace{3cm}}\text{ ft}^{-1}}$$

$$\boxed{r_H = 4.98\times10^{-4}\text{ ft}}$$

Example 3.1-4: Pressure Drop and Flow of Gases in Packed Bed

A water-gas shift reactor in a distributed-scale hydrogen plant is producing hydrogen at a rate of $65.2\frac{\text{lb}}{\text{h}}$. The reactor consists of a tubular packed bed of 5.25 cm diameter with 31.4 kg of a catalyst with a density of $76\frac{\text{lb}}{\text{ft}^3}$.

The void fraction of the bed is 0.57 and the spherical catalyst pellets have a diameter of 0.1 cm.

Determine the pressure drop of the reacting synthesis gas in the packed bed. The synthesis gas is entering the reactor at a pressure of 2066 kPa and has the following properties:

$$\mu = 0.048\frac{\text{lb}}{\text{ft}\cdot\text{h}}$$

$$\rho_0 = 0.356\frac{\text{lb}}{\text{ft}^3}$$

The differential form of the pressure drop in a packed bed reactor is given by the Ergun equation:

$$\frac{dP}{dz} = -\frac{G}{\rho g_c D_p}\left(\frac{1-\phi}{\phi^3}\right)\left(\frac{150(1-\phi)\mu}{D_p} + 1.75G\right)$$

The solution to this differential equation is given by:

$$\frac{P}{P_o} = (1-\alpha W)^{1/2}$$

where:

$$\alpha = \frac{2\beta_o}{A_c(1-\phi)\rho_c P_o}$$

$$\beta_o = \frac{G(1-\phi)}{\rho_o g_c D_p \phi^3}\left[\frac{150(1-\phi)\mu}{D_p} + 1.75G\right]$$

The first term in the brackets in the equation for β_o is dominant for laminar flow and the second term is dominant for turbulent flow. In these equations, the following notation and units are used:

P = Pressure $\left(\frac{\text{lb}}{\text{ft}^2}\right)$

ϕ = Void fraction (dimensionless)

g_c = Gravitational constant = $4.17 \times 10^8 \dfrac{\text{lb} \cdot \text{ft}}{\text{h}^2 \cdot \text{lb}_f}$

D_p = Diameter of particle in the bed (ft)

μ = Viscosity of gas $\left(\dfrac{\text{lb}}{\text{ft} \cdot \text{h}}\right)$

ρ_0 = Gas density $\left(\dfrac{\text{lb}}{\text{ft}^3}\right)$

ρ_c = Density of the catalyst $\left(\dfrac{\text{lb}}{\text{ft}^3}\right)$

$G = \dfrac{\dot{m}}{A}$ = Mass flux of synthesis gas $\left(\dfrac{\text{lb}}{\text{ft}^2 \cdot \text{h}}\right)$

$\dot{m}$ = Mass flow rate of synthesis gas $\left(\dfrac{\text{lb}}{\text{h}}\right)$

υ = Velocity of the gas in the reactor $\left(\dfrac{\text{lb}}{\text{h}}\right)$

z = Distance down packed bed (ft)

A = Cross – sectional area of the reactor (ft^2)

W = Mass of catalyst in the reactor (lb)

Strategy

The Ergun equation can be used for calculating the pressure at the outlet of the packed bed.

Solution

First we determine the value of β_o. All of the terms in the problem statement are in the appropriate units except for the particle diameter. We have:

$$D_p = 0.1\,\text{cm}\left(\frac{1\text{ ft}}{30.48\,\text{cm}}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ ft}$$

Another value we need to calculate to calculate β_o is the mass flux of synthesis gas G, given by:

$$G = \frac{\dot{m}}{A}$$

Where the cross sectional area is obtained as follows:

$$A = \frac{\pi(\_\_\_\_\_\_\_\_\ \text{cm})^2}{4}\left(\frac{1\ \text{ft}^2}{\_\_\_\_\_\_\_\_\_\_\ \text{cm}^2}\right) = \_\_\_\_\_\_\_\_\_\_\ \text{ft}^2$$

Entering this value and the mass flow rate into the equation for the mass flux G we get:

$$G = \frac{65.2\dfrac{\text{lb}}{\text{h}}}{\_\_\_\_\_\_\_\_\_\_\ \text{ft}^2} = \_\_\_\_\_\_\_\_\_\_\ \frac{\text{lb}}{\text{ft}^2\cdot\text{h}}$$

Substituting this and the other values into the equation for β_o :

$$\beta_o = \frac{G(1-\phi)}{\rho_o g_c D_p \phi^3}\left[\frac{150(1-\phi)\mu}{D_p} + 1.75G\right]$$

$$\beta_o = \frac{\_\_\_\_\_\_\_\_\_\_\ \dfrac{\text{lb}}{\text{ft}^2\cdot\text{h}}(1-0.57)}{0.356\dfrac{\text{lb}}{\text{ft}^3}\left(\_\_\_\_\_\_\_\_\_\_\ \dfrac{\text{lb}\cdot\text{ft}}{\text{h}^2\cdot\text{lb}_f}\right)(\_\_\_\_\_\_\_\_\ \text{ft})(0.57)^3}\left[\frac{150(1-0.57)\left(0.048\dfrac{\text{lb}}{\text{ft}\cdot\text{h}}\right)}{\_\_\_\_\_\_\_\_\_\_\ \text{ft}}\right.$$

$$\left. +1.75\left(\_\_\_\_\_\_\_\_\_\_\ \frac{\text{lb}}{\text{ft}^2\cdot\text{h}}\right)\right]$$

$$\beta_o = 77.91\frac{\text{lb}_f}{\text{ft}^3}$$

Note that this term has units of pressure $\left(\dfrac{\text{lb}_f}{\text{ft}^2}\right)$ per unit length (ft). It is also noted that the second term in the brackets is dominant, suggesting turbulent flow in this industrial reactor.

Now we need to determine the value of αW which is needed in the formula for the pressure drop. We first need to obtain the feed pressure, and catalyst weight in the appropriate units.

The feed pressure in $\dfrac{\text{lb}_f}{\text{ft}^2}$ is:

$$P_o = 2066\ \text{kPa}\left(\frac{\_\_\_\_\_\_\_\_\ \dfrac{\text{lb}_f}{\text{in}^2}}{\_\_\_\_\_\_\_\_\ \text{kPa}}\right)\left(\frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}\right) = \_\_\_\_\_\_\_\_\_\_\ \frac{\text{lb}_f}{\text{ft}^2}$$

The catalyst weight W in lb_m is:

$$W = 31.4\ \text{kg}\left(\frac{1\ \text{lb}}{0.454\,\text{kg}}\right) = \underline{\qquad\qquad}\ \text{lb}$$

Thus,

$$\alpha W = \frac{2\beta_o W}{A_c(1-\phi)\rho_c P_o} = \frac{2\left(77.91\dfrac{\text{lb}_f}{\text{ft}^3}\right)(\underline{\qquad\qquad}\ \text{lb})}{\underline{\qquad\qquad}\ \text{ft}^2(1-0.57)\left(76\dfrac{\text{lb}}{\text{ft}^3}\right)\left(\underline{\qquad\qquad}\ \dfrac{\text{lb}_f}{\text{ft}^2}\right)} = \underline{\qquad\qquad}$$

The exit pressure can be determined by entering this value into the equation for the pressure ratio $\frac{P}{P_o}$:

$$\frac{P}{P_o} = (1-\alpha W)^{1/2}$$

Substituting the values of αW and the pressure at the entrance of the packed bed P_0 yields:

$$\frac{P}{\left(\underline{\qquad\qquad}\ \dfrac{\text{lb}_f}{\text{ft}^2}\right)} = (\underline{\qquad\qquad})^{1/2}$$

Solving for the pressure at the outlet P, we get:

$$P = (\underline{\qquad\qquad})^{1/2}\left(\underline{\qquad\qquad}\ \frac{\text{lb}_f}{\text{ft}^2}\right) = 35373\frac{\text{lb}_f}{\text{ft}^2}$$

Hence, the pressure drop is given by:

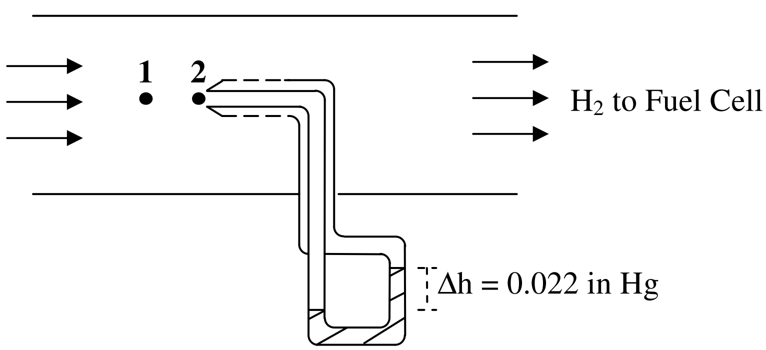
$$\Delta P = P - P_0 = \left(35373\frac{\text{lb}_f}{\text{ft}^2} - \underline{\qquad\qquad}\ \frac{\text{lb}_f}{\text{ft}^2}\right)\left(\frac{\underline{\qquad\qquad}}{\underline{\qquad\qquad}}\right)$$

$$\boxed{\Delta P = \underline{\qquad\qquad}\ \text{psi}}$$

Example 3.2-1: Flow Measurement using a Pitot Tube

A Pitot Tube is used for measuring the flow in a circular pipe. To determine the flow of hydrogen at room temperature to a proton-exchange membrane fuel cell, a Pitot tube with a coefficient of 0.84 is used in a pipe with a diameter of 1". The static-pressure of hydrogen is measured to be 12.34 mm of Hg above atmospheric pressure.

Determine the maximum and average velocities and flow rates of hydrogen in the pipe if the reading on the manometer is 0.022 in of Hg. The following figure shows the diagram of Pitot Tube for this problem.



Strategy

To solve this problem we need to obtain the velocity value from Bernoulli Equation applied to a Pitot Tube.

Solution

The Bernoulli equation applied to a Pitot Tube is given by:

$$\upsilon_1 = C_p \sqrt{\frac{2(\underline{\qquad\qquad\qquad})}{\underline{\qquad\qquad}}}$$

where C_p is the value of the Pitot tube coefficient.

The viscosity of hydrogen can be obtained from Appendix A.3 of Geankoplis to be.

$$\mu = \underline{\qquad\qquad\qquad} \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

The density can be calculated using Ideal Gas equation of state as shown in the following steps:

$$\rho_{H_2 \text{ @ P = 1 atm}} = \frac{\underline{\qquad\qquad}}{\text{RT}} = \frac{(\underline{\quad\quad} \text{ atm})\left(\underline{\qquad\qquad} \frac{\text{kg}}{\text{mol}}\right)}{\left(8.206\times10^{-5} \frac{\underline{\quad\quad}\cdot\underline{\quad\quad}}{\underline{\quad\quad}\cdot\underline{\quad\quad}}\right)(298.15 \text{ K})}$$

$$\rho_{H_2\ @\ P = 1\ \text{atm}} = \underline{\hspace{4cm}} \frac{\text{kg}}{\text{m}^3}$$

First we need to convert the manometric static pressure to absolute pressure. To do this, we will use the density of mercury as $13533.61 \frac{\text{kg}}{\text{m}^3}$, obtained from Table 2-31 of *Perry's Chemical Engineers' Handbook*, 8th Edition. Thus,

$$\Delta P_{\text{static}} = \left(\rho_{Hg} - \rho_{H_2\ @\ P = 1\ \text{atm}}\right) gh$$

$$\Delta P_{\text{static}} = \left(13533.61 \frac{\text{kg}}{\text{m}^3} - \underline{\hspace{4cm}} \frac{\text{kg}}{\text{m}^3}\right)\left(\underline{\hspace{4.5cm}} \frac{\text{m}}{\text{s}^2}\right)(\underline{\hspace{5cm}}\ \text{m Hg})$$

$$\Delta P_{\text{static}} = 1638\ \text{Pa}$$

We can add this value to the atmospheric pressure of 101325 Pa to get the absolute pressure to be ______________ Pa. Since the density of hydrogen was calculated at atmospheric pressure, we need to correct the density value for the actual pressure of hydrogen in the pipe. This can be done by multiplying the density at 1 atm of pressure by the pressure ratio. Therefore,

$$\rho_{H_2} = \rho_{H_2\ @\ P = 1\ \text{atm}} \left(\frac{P}{P_{\text{atm}}}\right) = \underline{\hspace{4cm}} \frac{\text{kg}}{\text{m}^3}\left(\frac{\underline{\hspace{4cm}}\ \text{Pa}}{101325\ \text{Pa}}\right)$$

$$\rho_{H_2} = \underline{\hspace{4cm}} \frac{\text{kg}}{\text{m}^3}$$

Now we can calculate the pressure difference using the change in the height of mercury in the manometer as shown below:

$$\Delta P = \left(\rho_{Hg} - \rho_{H_2}\right) g\Delta h$$

$$\Delta P = \left(13533.61 \frac{\text{kg}}{\text{m}^3} - \underline{\hspace{4cm}} \frac{\text{kg}}{\text{m}^3}\right)\left(\underline{\hspace{5cm}} \frac{\text{m}}{\text{s}^2}\right)(\underline{\hspace{4cm}}\ \text{in Hg})\left(\frac{0.0254\ \text{m Hg}}{1\ \text{in Hg}}\right)$$

$$\Delta P = \underline{\hspace{4cm}}\ \text{Pa}$$

Substituting this pressure drop into the equation for the velocity of hydrogen in the tube yields:

$$\upsilon_1 = 0.84 \sqrt{\frac{2(\underline{\hspace{4cm}}\ \text{Pa})}{\underline{\hspace{4cm}} \frac{\text{kg}}{\text{m}^3}}}$$

$$\upsilon_1 = \_\_\_\_\_\_\_\_\_\_ \frac{\text{m}}{\text{s}}$$

Since the point 1 in the tube is at the center of the tube, where the velocity reaches its maximum value, the velocity we calculated is the maximum velocity.

$$\boxed{\upsilon_{max} = \_\_\_\_\_\_\_\_\_\_ \frac{\text{m}}{\text{s}}}$$

The average velocity of a fluid in a pipe can be estimated using Figure 2.10-2 of Geankoplis as a function of Reynolds number. The value of Reynolds number using the maximum velocity for this problem is given by:

$$\text{Re} = \frac{D\upsilon_{max}\rho_{H_2}}{\mu} = \frac{(1\text{ in})\left(\frac{0.0254\text{ m}}{1\text{ in}}\right)\left(\_\_\_\_\_\_\_\_\_\_ \frac{\text{m}}{\text{s}}\right)\left(\_\_\_\_\_\_\_\_\_\_ \frac{\text{kg}}{\text{m}^3}\right)}{\_\_\_\_\_\_\_\_\_\_ \frac{\text{kg}}{\text{m}\cdot\text{s}}}$$

$$\text{Re} = 8507$$

Locating this value in Figure 2.10-2 we can estimate the ratio of the average velocity to the maximum velocity to be:

$$\frac{\upsilon_{av}}{\upsilon_{max}} \approx \_\_\_\_\_\_\_\_\_\_$$

Solving for the average velocity and entering the value of the maximum velocity into this equation we get:

$$\upsilon_{av} \approx \_\_\_\_\_\_\_\_\_\_ (\upsilon_{max}) \approx \_\_\_\_\_\_\_\_\_\_ \left(\_\_\_\_\_\_\_\_\_\_ \frac{\text{m}}{\text{s}}\right)$$

$$\boxed{\upsilon_{av} \approx 27 \frac{\text{m}}{\text{s}}}$$

To calculate the average and maximum flow rates of hydrogen we need to multiply the corresponding velocity value by the cross-sectional area of the pipe. The area of the pipe is calculated as shown below:

$$A = \frac{\pi D^2}{4} = \frac{\pi(1\text{ in})^2}{4}\left(\frac{\_\_\_\_\_\_\_\_\_\_ \text{m}^2}{1\text{ in}^2}\right) = \_\_\_\_\_\_\_\_\_\_ \text{m}^2$$

Multiplying the cross-sectional area by both velocity values we get:

$$\dot{V}_{av} = \underline{\hspace{4cm}}\ m^2\left(27\frac{m}{s}\right)$$

$$\boxed{\dot{V}_{av} = \underline{\hspace{4cm}}\ \frac{m^3}{s}}$$

$$\dot{V}_{max} = \underline{\hspace{4cm}}\ m^2\left(\underline{\hspace{2cm}}\ \frac{m}{s}\right)$$

$$\boxed{\dot{V}_{max} = \underline{\hspace{4cm}}\ \frac{m^3}{s}}$$

Example 3.3-1: NPSH Available for Pump

Water at a temperature of 25°C is being pumped to a boiler in a distributed-scale steam-methane reforming plant. The water is entering the boiler through a commercial steel pipe with a diameter of 0.1 m and a length of 25 m. The estimated friction factor for the flow conditions in this process is estimated to be 0.0045.

Determine the available net positive suction head (NPSH) of the pump if the velocity of the water in the pipe is of $2.3\frac{\text{m}}{\text{s}}$

Strategy

This problem can be solved using the equation for the NPSH as a function of the pressure and thermodynamic properties of the fluid in the pipe.

Solution

The following equation shows the relation between the NPSH available and the conditions in the system:

$$g\left(\text{NPSH}\right)_A = \frac{P_1 - P_{vp}}{\rho} + gz_1 - \frac{\upsilon^2}{2} - \sum F$$

where:

g = Acceleration due to gravitational force = $9.80665\frac{\text{m}}{\text{s}^2}$

P_1 = Pressure of the fluid before entering the pump (Pa)

P_{vp} = Saturation pressure of fluid at the process temperature (Pa)

ρ = Density of fluid $\left(\frac{\text{kg}}{\text{m}^3}\right)$

z_1 = Difference in height between the pump and the point at pressure P_1 (m)

υ = Velocity of the fluid $\left(\frac{\text{m}}{\text{s}}\right)$

$\sum F$ = Friction loss in suction line to pump $\left(\frac{\text{J}}{\text{kg}}\right)$

Since there is no difference in the height between the pipe and the pump, the term gz_1 in the equation for $g(\text{NSPH})_A$ is neglected in this problem. Thus,

$$g\left(\text{NPSH}\right)_A = \frac{P_1 - P_{vp}}{\rho} - \frac{\upsilon^2}{2} - \sum F$$

The density and saturation pressure of water at 25 °C can be found in Geankoplis in Tables A.2-3 and A.2-9 respectively:

$$\rho = \underline{\hspace{3cm}} \frac{\text{kg}}{\text{m}^3}$$

$$P_{vp} = \underline{\hspace{2.5cm}} \text{ Pa}$$

The only remaining unknown value in the equation for NSPH_A is the friction loss, which can be calculated using the following equation:

$$F = 4f \frac{\Delta L}{D} \frac{\upsilon^2}{2}$$

Substituting the corresponding values into this equation, we get:

$$\sum F = 4\left(\underline{\hspace{2.5cm}}\right) \frac{25 \text{ m}}{0.1 \text{ m}} \frac{\left(2.3 \frac{\text{m}}{\text{s}}\right)^2}{2} = \underline{\hspace{2cm}} \frac{\text{J}}{\text{kg}}$$

Now we can enter all the known quantities into the equation for NSPH to yield:

$$\left(9.80665 \frac{\text{m}}{\text{s}^2}\right)\left(\text{NPSH}\right)_A = \frac{\left(101325 \text{ Pa}\right) - \left(\underline{\hspace{2.5cm}} \text{ Pa}\right)}{\underline{\hspace{2.5cm}} \frac{\text{kg}}{\text{m}^3}} - \frac{\left(2.3 \frac{\text{m}}{\text{s}}\right)^2}{2} - 11.9 \frac{\text{J}}{\text{kg}}$$

Solving for the NPSH_A:

$$\left(\text{NPSH}\right)_A = \left[\frac{\left(101325 \text{ Pa}\right) - \left(\underline{\hspace{2.5cm}} \text{ Pa}\right)}{\underline{\hspace{2.5cm}} \frac{\text{kg}}{\text{m}^3}} - \frac{\left(2.3 \frac{\text{m}}{\text{s}}\right)^2}{2} - 11.9 \frac{\text{J}}{\text{kg}}\right] \frac{1}{\left(9.80665 \frac{\text{m}}{\text{s}^2}\right)}$$

$$\boxed{\left(\text{NPSH}\right)_A = \underline{\hspace{2.5cm}} \text{ m}}$$

Example 3.3-2: Calculation of Brake Horsepower of a Pump

Determine the brake horsepower of the pump from Example 3.3-1 operating at a flow rate of $683.1\frac{\text{kg}}{\text{hr}}$ for feeding water to a boiler in a Steam-Methane reforming plant. Assume the characteristic curves of this pump are described by Figure 3.3-3 of Geankoplis.

Strategy

The brake horsepower of the pump described in this example can be calculated using the mass flow rate of fluid and the work performed by the pump.

Solution

The brake hp of a pump can be calculated using Equation 3.3-2 of Geankoplis:

$$\text{brake hp} = \frac{-W_S \dot{m}}{550\eta}$$

where:

W_S = Work performed by the pump $\left(\frac{\text{ft} \cdot \text{lb}_f}{\text{lb}}\right)$

$\dot{m}$ = Mass flow rate of fluid $\left(\frac{\text{lb}}{\text{s}}\right)$

η = Efficiency of the pump

First we need to calculate the work performed by the pump, defined as follows:

$$W_s = -H\frac{g}{g_C}$$

In this equation, the value of the head of fluid H is unknown. However, it can be read from the characteristic curves of the pump, shown in Figure 3.3-3. The volumetric flow rate of water required for using Figure 3.3-3 is calculated by multiplying the mass flow rate by the density of water at the process conditions as shown below:

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{683.1\frac{\text{kg}}{\text{hr}}\left(\frac{\rule{2cm}{0.4pt}}{\rule{2cm}{0.4pt}}\right)}{997.08\frac{\text{kg}}{\text{m}^3}\left(\frac{\rule{4cm}{0.4pt}\ \text{m}^3}{\text{ft}^3}\right)}$$

$$\dot{V} = \_\_\_\_\_\_\_\_ \frac{ft^3}{min}\left(\frac{\_\_\_\_\_\_\_\_ \ gal}{1\ ft^3}\right) = \_\_\_\_\_\_\_\_ \frac{gal}{min}$$

With this volumetric flow rate, we can estimate the head of fluid and the efficiency of the pump from Figure 3.3-3 to be:

$$H \approx \_\_\_\_\_\_\_\_ \ ft$$

$$\eta \approx \_\_\_\_\_\_\_\_$$

We can substitute the head of fluid value into the equation for the work W_S to yield:

$$W_S = -\_\_\_\_\_\_ \ ft\left(\frac{\_\_\_\_\_\_\_\_ \frac{ft}{hr^2}}{\_\_\_\_\_\_\_\_ \frac{lb \cdot ft}{hr^2 \cdot lb_f}}\right)$$

$$W_S = \_\_\_\_\_\_\_\_ \ \frac{lb_f \cdot ft}{lb}$$

The negative value of the work W_S indicates that the fluid is performing work over the pump. Now we can enter the values we calculated into the equation for the brake horsepower of the pump to get:

$$\text{brake hp} = \frac{-\left(\_\_\_\_\_\_\_\_ \ \frac{lb_f \cdot ft}{lb}\right)\left(683.1\frac{kg}{hr}\right)\left(\frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}\right)\left(\frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}\right)}{550(\_\_\_\_\_\_)}$$

$$\boxed{\text{brake hp} = \_\_\_\_\_\_ \ hp}$$

Example 3.3-3: Brake-kW Power of a Centrifugal Fan

Air at a flow rate of $2.90\frac{m^3}{min}$ (measured at 1 atm and 298.15 K) and a velocity of $67.1\frac{m}{s}$ enters a proton-exchange membrane fuel cell stack through a centrifugal fan. The air is entering the fan at a pressure of 1.009 bar and a temperature of 100°F. What is the discharge pressure of the air if the fan has a brake power of 2.12 kW and an efficiency of 70%?

Strategy

This problem can be solved by performing a mechanical energy balance on the system, and using the definition of brake power of a centrifugal fan.

Solution

The mechanical-energy balance for this problem ($z_1 = z_2$, $\upsilon_{in} = 0$, $\sum F = 0$) is given by:

$$W_S = \frac{P_1 - P_2}{\rho} - \frac{(\upsilon_{out})^2}{2}$$

W_S = Work performed by the centrifugal fan $\left(\frac{J}{kg}\right)$

P_1 = Pressure of air at the suction point (Pa)

P_2 = Pressure of air at the discharge point (Pa)

ρ = Average density of the air $\left(\frac{kg}{m^3}\right)$

υ_{out} = Velocity of air at the discharge point $\left(\frac{m}{s}\right)$

Before being able to calculate the discharge pressure from this equation, we need to calculate the velocity of air in the outlet, the average density and the work performed by the fan on the fluid.

The density of the air entering the fan is calculated using the ideal gas law and the properties of an ideal gas at standard conditions. In the following calculation, the sub-index 1 represents the conditions of the air at the inlet of the fan.

$$\rho_1 = \frac{M_{air}}{\hat{V}_{std}}\left(\frac{T_{std}}{T_1}\right)\left(\frac{P_1}{P_{std}}\right) = \frac{\underline{\qquad\qquad} \frac{kg\ air}{kmol}}{\underline{\qquad\qquad} \frac{m^3}{kmol}}\left(\frac{\underline{\qquad\qquad}\ K}{311.15\ K}\right)\left(\frac{1.009\ bar}{\underline{\qquad\qquad}\ bar}\right)$$

$$\rho_1 = \underline{\hspace{3cm}} \frac{kg}{m^3}$$

The density of the air leaving through the fan can be determined using the pressure ratio between the suction and discharge points. After substituting the known pressure and density we get:

$$\rho_2 = \rho_1\left(\frac{P_2}{P_1}\right) = \underline{\hspace{3cm}}\frac{kg}{m^3}\left(\frac{P_2}{1.009 \text{ bar}}\right)\left(\frac{1 \text{ bar}}{\underline{\hspace{3cm}} \text{ Pa}}\right)$$

$$\rho_2 = (\underline{\hspace{4cm}})P_2$$

Now we can obtain the average density of the air as a function of the discharge pressure P_2:

$$\rho = \frac{\rho_1 + \rho_2}{2} = \frac{\underline{\hspace{2.5cm}}\frac{kg}{m^3} + (\underline{\hspace{4cm}})P_2}{2}$$

$$\rho\left(\frac{kg}{m^3}\right) = \underline{\hspace{2cm}} + (\underline{\hspace{3cm}})P_2$$

Substituting this expression into the mechanical-energy balance equation:

$$W_S = \frac{P_1 - P_2}{\underline{\hspace{2cm}} + (\underline{\hspace{3cm}})P_2} - \frac{(\upsilon_{out})^2}{2}$$

Now we need to calculate the left hand side of this equation. Equation 3.3-2 of Geankoplis defines the brake power of a centrifugal fan as follows:

$$\text{brake kW} = \frac{-W_S \dot{m}}{1000\eta}$$

where:

$\dot{m}$ = Mass flow rate of fluid $\left(\frac{kg}{s}\right)$

η = Efficiency of the fan

We can solve for the work W_S performed by the fan:

$$W_S = \underline{\hspace{3.5cm}}\left(\frac{\underline{\hspace{2cm}}}{\underline{\hspace{2cm}}}\right)$$

The value of the mass flow rate $\dot{m}$ can be obtained from the properties of an ideal gas at standard conditions. Thus,

$$\dot{m} = \frac{\dot{V}\_\_\_\_\_\_}{\hat{V}}\left(\frac{T_{std}\_\_\_\_\_\_}{\_\_\_\_\_\_}\right)$$

The sub-index 0 of the temperature indicates the temperature at which the air flow rate was measured. Substituting the corresponding numeric values into this equation yields:

$$\dot{m} = \frac{\_\_\_\_\_\_\frac{m^3}{min}\left(\frac{1\ min}{60\ s}\right)\left(\_\_\_\_\_\_\frac{kg}{kmol}\right)}{\left(\_\_\_\_\_\_\frac{m^3}{kmol}\right)}\left(\frac{273.15\ K}{\_\_\_\_\_\_\ K}\right)$$

$$\dot{m} = \_\_\_\_\_\_\ \frac{kg}{s}$$

Entering this value and the brake power of the fan into the equation for the work W_S:

$$W_s = -2.12\ kW\left(\frac{1000(0.70)}{\_\_\_\_\_\_\ \frac{kg}{s}}\right) = -\_\_\_\_\_\_\ \frac{J}{kg}$$

Now, after substituting the calculated values, the energy balance equation is given by:

$$-\_\_\_\_\_\_\ \frac{J}{kg} = \frac{1.009\ bar\left(\frac{1\times10^5\ Pa}{1\ bar}\right) - P_2}{\_\_\_\_ + (\_\_\_\_\_\_)P_2} - \frac{\left(67.1\frac{m}{s}\right)^2}{2}$$

$$\_\_\_\_\_\_\ \frac{J}{kg} = \frac{1.009\times10^5\ Pa - P_2}{\_\_\_\_ + (\_\_\_\_\_\_)P_2}$$

This equation can be solved for P_2 as shown in the following steps:

$$\_\_\_\_\_\_\ \frac{J}{kg}\left[\_\_\_\_ + (\_\_\_\_\_\_)P_2\right] = 1.009\times10^5\ Pa - P_2$$

$$\_\_\_\_\_\_ + (\_\_\_\_\_\_)P_2 = 1.009\times10^5\ Pa - P_2$$

$$\_\_\_\_\_\_\ P_2 + P_2 = \left(1.009\times10^5\right) + 13386.4$$

$$P_2 = \frac{\_\_\_\_\_\_}{0.8673}$$

$$\boxed{P_2 = \_\_\_\_\_\_\ Pa\left(\frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}\right) = 1.318\ bar}$$

Example 3.3-4: Compression of Methane

A compressor in a steam-methane reforming process for hydrogen production is compressing natural gas at room temperature from a pressure of 1 atm to 21.8 atm before entering the reforming reactor. What percent of power is saved when operating the compressor at isothermal conditions if compared to adiabatic compression?

Strategy

The equation for calculating the brake power of a compressor allows us to compare both adiabatic and isothermal compression processes.

Solution

The equation for calculating the power required by the compressor is shown below:

$$\text{brake kW} = \frac{-W_S \dot{m}}{\eta}$$

The value of the work W_S in this equation is in kJ. The values of the mass flow rate $\dot{m}$ and the efficiency η are not given in the problem statement. However, for calculating the amount of power saved, the values of $\dot{m}$ and η are not required, as it will be shown in the following steps. We need to calculate the work performed by the compressor at both adiabatic and isothermal conditions.

For adiabatic compression, the work W_S is given by:

$$-W_{S,\text{adiabatic}} = \frac{\gamma}{\gamma - 1} \frac{RT_1}{M} \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

The parameter γ represents the heat capacity ratio. For methane, a value of $\gamma = 1.31$ is given in Geankoplis. Hence, the work for adiabatic compression is calculated as follows:

$$-W_{S,\text{adiabatic}} = \left(\frac{\rule{3cm}{0.4pt}}{\rule{3cm}{0.4pt}} \right) \left(\frac{\rule{3cm}{0.4pt} \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} (298.15 \text{ K})}{\rule{1.5cm}{0.4pt} \frac{\text{kg}}{\text{kmol}}} \right) \left[\left(\frac{21.8 \text{ atm}}{1 \text{ atm}} \right)^{\frac{\rule{2cm}{0.4pt}}{\rule{2cm}{0.4pt}}} - 1 \right]$$

$$-W_{S,\text{adiabatic}} = 702.9 \frac{\text{kJ}}{\text{kg}}$$

For isothermal operation, the work W_S is calculated using the following equation:

$$-W_{S,\ isothermal} = \frac{\underline{\qquad\qquad\qquad\qquad}}{M} \log \frac{P_2}{P_1}$$

Substituting the pressure ratio into this equation and the temperature of methane before entering the compressor yields:

$$-W_{S,\ isothermal} = \underline{\qquad\qquad\qquad} \frac{kJ}{kg}$$

The percent of power saved can be calculated dividing the difference in power between adiabatic and isothermal compression by the power required for adiabatic compression. Thus,

$$\text{Power saved } (\%) = \frac{(\text{brake kW})_{\text{adiabatic}} - (\text{brake kW})_{\text{isothermal}}}{(\text{brake kW})_{\text{adiabatic}}} \times 100$$

Substituting the equations for the power required by the compressor and the work values in this equation, we get:

$$\text{Power saved } (\%) = \frac{\cancel{\frac{\dot{m}}{\eta}}\left[-W_{S,adiabatic} - \left(-W_{S,isothermal}\right)\right]}{\cancel{\frac{\dot{m}}{\eta}}\left(-W_{S,adiabatic}\right)} \times 100$$

$$\text{Power saved } (\%) = \left(\frac{\underline{\qquad\qquad\qquad} \frac{kJ}{kg} - \underline{\qquad\qquad\qquad} \frac{kJ}{kg}}{\underline{\qquad\qquad\qquad} \frac{kJ}{kg}} \right) \times 100$$

$$\boxed{\text{Power saved} = \underline{\qquad\qquad\qquad}}$$

Example 3.8-3: Laminar Flow in a Circular Tube

An aqueous solution of 40 % methanol is flowing from the fuel reservoir to a stack of direct-methanol fuel cells in a mobile phone through a pipe with an inner diameter of 3 mm and a length of 2 cm. Use the equation derived in Geankoplis for laminar flow in a circular tube to determine the pressure drop along the pipe if the maximum Reynolds number for methanol in the pipe is 1850.

Strategy

We can use the Hagen-Poiseuille equation to calculate the pressure drop between the methanol reservoir and the fuel cell stack.

Solution

The Hagen-Poiseuille equation is derived in Geankoplis for laminar flow in a circular tube, shown below:

$$P_1 - P_2 = \frac{32\mu\upsilon_{av}L}{D^2}$$

The viscosity of the methanol solution at 25 ºC can be obtained from Figure A.3-4 to be:

$$\mu = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{kg}{m \cdot s}$$

The only parameter we need to calculate before being able to determine the pressure drop is the average velocity. We can obtain the ratio of the average velocity to the maximum velocity from Figure 2.10-2 of Geankoplis at the Reynolds number of 1850. From this figure, it can be seen that for laminar flow the ratio of the velocities remains constant and equal to __________. Thus,

$$\upsilon_{av} = \_\_\_\_\_\_\_\_ \upsilon_{max}$$

From the definition of Reynolds number we can solve for the maximum velocity to yield:

$$\upsilon_{max} = \frac{\_\_\_\_\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}$$

The density of an aqueous solution of 40 % methanol can be obtained from Table 2-109 of *Perry's Chemical Engineers' Handbook*, 7th Edition to be:

$$\rho_{@\,20^{\circ}C} = 934.5 \frac{kg}{m^3}$$

Substituting the corresponding quantities into the definition of Reynolds number, we get:

$$\upsilon_{max} = \frac{1850\left(\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{kg}{m \cdot s}\right)}{\left(3\times10^{-3}\,m\right)\left(\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{kg}{m^3}\right)} = \_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{m}{s}$$

With this value, we can determine the average velocity from the data read from Figure 2.10-2:

$$\upsilon_{av} = \_\_\_\_\_\_\_\_\_\_\_\_\_\left(\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{m}{s}\right) = 0.6\frac{m}{s}$$

Finally, after entering this velocity and the rest of the values into the Hagen-Poiseuille equation we can determine the pressure drop as shown in the following steps:

$$P_1 - P_2 = \frac{32\left(\_\_\_\_\_\_\_\_\_\_\_\_\_ \frac{kg}{m \cdot s}\right)\left(0.6\frac{m}{s}\right)(\_\_\_\_\_\_\_\_\_\_\_\_\_\ m)}{\left(3\times10^{-3}\,m\right)^2}$$

$$\boxed{P_1 - P_2 = \_\_\_\_\_\_\ Pa}$$

Example 3.11-1: Dimensionless Groups

Using dimensionless numbers determine which of the following forces is the most dominant for the flow conditions in a polymer-electrolyte membrane fuel cell and direct-methanol fuel cell:

-Inertia Force

-Gravity Force

-Pressure Force

-Viscous Force

The flow conditions in both fuel cells are given in the following set of data. The direct-methanol fuel cell is using an aqueous solution of 40 % methanol as fuel.

Polymer-Electrolyte Membrane Fuel Cell

Pressure: 2.5 atm

Temperature: 80 ºC

Velocity of hydrogen: $15\,\frac{\mathrm{m}}{\mathrm{s}}$

Hydraulic diameter of the channel: $1\times10^{-3}\,\mathrm{m}$

Direct Methanol Fuel Cell

Pressure: 1 atm

Temperature: 25 ºC

Velocity of methanol: $0.49\,\frac{\mathrm{m}}{\mathrm{s}}$

Density: $931.5\,\frac{\mathrm{kg}}{\mathrm{m}^3}$

Hydraulic diameter of the channel: $8.57\times10^{-4}\,\mathrm{m}$

Strategy

The Froude, Euler and Reynolds numbers establish a relation between the forces given in the problem statement and hence can be used for determining the most dominant force for a specified flow conditions.

Solution

The definitions of the dimensionless groups that will be used for solving this problem are given in the following equations.

$$N_{Fr} = \frac{\text{inertia force}}{\text{gravity force}} = \frac{\upsilon^2}{gL}$$

$$N_{Eu} = \frac{\text{pressure force}}{\text{inertia force}} = \frac{\underline{\hspace{1.5cm}}}{\underline{\hspace{1.5cm}}}$$

$$N_{Re} = \frac{\text{inertia force}}{\text{viscous force}} = \frac{L\upsilon\rho}{\mu}$$

In these equations the parameter L represents the characteristic length. For this problem, the characteristic length will be given by the hydraulic diameter of the channel D_H. Thus,

$$N_{Fr} = \frac{\text{inertia force}}{\text{gravity force}} = \frac{\upsilon^2}{gD_H}$$

$$N_{Eu} = \frac{\text{pressure force}}{\text{inertia force}} = \frac{\underline{\hspace{1.5cm}}}{\underline{\hspace{1.5cm}}}$$

$$N_{Re} = \frac{\text{inertia force}}{\text{viscous force}} = \frac{\underline{\hspace{2.5cm}}}{\underline{\hspace{2.5cm}}}$$

For comparison of the forces involved in these dimensionless numbers, we will use the reciprocal of the Euler number. By doing this, we can compare the three numbers using the inertia force as reference. Therefore,

$$\frac{1}{N_{Eu}} = \frac{\text{inertia force}}{\text{pressure force}} = \frac{\underline{\hspace{2cm}}}{\underline{\hspace{1.2cm}}}$$

The viscosity of the methanol solution at room temperature is obtained from Appendix A.3 to be:

$$\mu = \underline{\hspace{4cm}} \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

For hydrogen at a temperature of 80 ºC, the density and viscosity obtained from Table 2-223 of *Perry's Chemical Engineers' Handbook*, 8^{th} Edition and are shown below:

$$\mu = 9.9982 \times 10^{-6} \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

$$\rho = 0.178 \frac{\text{kg}}{\text{m}^3}$$

Now we can proceed to calculate the dimensionless numbers for the flow conditions in both types of fuel cell. For the polymer-electrolyte membrane, we can substitute the given parameters into the equations for Froude, Euler and Reynolds as shown in the following steps:

$$N_{Fr} = \frac{\upsilon^2}{gD_H} = \frac{\left(\_\_\_\_\_\_\_\ \frac{m}{s}\right)^2}{\left(9.80665\,\frac{m}{s^2}\right)(\_\_\_\_\_\_\_\_\_\_\ m)}$$

$$N_{Fr} = \_\_\_\_\_\_\_\_\_\_$$

$$\frac{1}{N_{Eu}} = \frac{\_\_\_\_\_\_\_\ \frac{kg}{m^3}\left(\_\_\_\_\_\_\_\ \frac{m}{s}\right)^2}{2.5\ \text{atm}\left(\frac{101325\ \text{Pa}}{1\ \text{atm}}\right)}$$

$$\frac{1}{N_{Eu}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$N_{Re} = \frac{(\_\_\_\_\_\_\_\_\_\_\ m)\left(\_\_\_\ \frac{m}{s}\right)\left(0.178\,\frac{kg}{m^3}\right)}{\_\_\_\_\_\_\_\_\_\_\_\_\_\ \frac{kg}{m \cdot s}}$$

$$N_{Re} = \_\_\_\_\_\_\_\_\_\_\_$$

Conclusion:

Repeating a similar procedure for the direct-methanol fuel cell, we find the following values for the dimensionless numbers:

$$N_{Fr} = \frac{\upsilon^2}{gD_H} = \frac{\left(\_\_\_\_\_\_\_\ \frac{m}{s}\right)^2}{\left(\_\_\_\_\_\_\_\ \frac{m}{s^2}\right)(\_\_\_\_\_\_\_\_\_\_\_\_\ m)}$$

$$N_{Fr} = \_\_\_\_\_\_\_$$

$$\frac{1}{N_{Eu}} = \frac{\underline{\hspace{3em}}\ \frac{kg}{m^3}\left(\underline{\hspace{3em}}\ \frac{m}{s}\right)^2}{\underline{\hspace{5em}}}$$

$$\frac{1}{N_{Eu}} = \underline{\hspace{6em}}$$

$$N_{Re} = \frac{\left(\underline{\hspace{5em}}\ m\right)\left(\underline{\hspace{3em}}\ \frac{m}{s}\right)\left(\underline{\hspace{3em}}\ \frac{kg}{m^3}\right)}{\underline{\hspace{5em}}\ \frac{kg}{m \cdot s}}$$

$$N_{Re} = \underline{\hspace{3em}}$$

Conclusion: