

Chapter 2

Principles of Momentum Transfer and Overall Balances

In fuel cells, the fuel is usually in gas or liquid phase. Thus, the student must be familiar with the principles of fluid mechanics or momentum transfer, which will be covered in the following problem modules. In later sections of this chapter, situations combining momentum transfer and heat transfer will be illustrated.

- 2.2-3 Conversion of Pressure to Head of a Fluid
- 2.3-1 Diffusivity of Hydrogen inside a Fuel Cell
- 2.5-1 Reynolds Number of Hydrogen flowing into a Fuel Cell
- 2.6-1 Flow of Hydrogen into Fuel Cells
- 2.6-3 Velocity of Hydrogen in Bipolar Plate Channel
- 2.7-1 Energy Balance on Ethanol boiler
- 2.10-1 Methanol Flow in Fuel Cell
- 2.10-2 Use of Friction Factor in Laminar Flow
- 2.10-3 Use of Friction Factor in Turbulent Flow
- 2.10-4 Trial-and-Error Solution to Calculate Pipe Diameter
- 2.10-5 Flow of Gas in Line and Pressure Drop
- 2.10-8 Entry Length for a Fluid in a Rectangular Channel
- 2.11-1 Compressible Flow of a Gas in a Pipe Line
- 2.11-2 Maximum Flow for Compressible Flow of a Gas

Example 2.2-3: Conversion of Pressure to Head of a Fluid

Hydrogen is stored in a compressed gas tank with a volume of 50 L at a pressure of 140 atm and a temperature of 25 °C. What is the pressure in the tank in mm Hg and inches of water?

Strategy

To determine the pressure of hydrogen as head of a fluid we need to use the equation for calculating pressure of a fluid as a function of height.

Solution

The equation for pressure as a function of height is shown below.

$$P = \rho gh$$

where:

ρ = density of fluid

g = acceleration due to gravitational force

h = head or height of fluid

We can solve this equation for the head h to yield:

$$h = \frac{P}{\rho g}$$

To determine the pressure in mm of Hg we need to use the density of mercury, which can be obtained from Table 2-31 *Perry's Chemical Engineers' Handbook*, 8th Edition to be:

$$\rho_{\text{Hg}} = \frac{\text{kg}}{\text{m}^3}$$

Since the units in the expression for the head of a fluid need to match, the value of g must be in $\left(\frac{\text{m}}{\text{s}^2}\right)$ and the pressure must be converted to Pa $\left(\frac{\text{N}}{\text{m}^2}\right)$. After entering the values into the equation for h we get:

$$h = \frac{140 \text{ atm} \left(\frac{\frac{\text{N}}{\text{m}^2}}{1 \text{ atm}} \right) \left(\frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{1 \text{ N}} \right)}{\left(\frac{\text{kg}}{\text{m}^3} \right) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}$$

$$h = \underline{\hspace{2cm}} \text{ m Hg}$$

This value can be converted to mm of Hg by multiplying by the conversion factor from m to mm:

$$h = \underline{\hspace{2cm}} \text{ m Hg} \left(\frac{\underline{\hspace{2cm}}}{\underline{\hspace{2cm}}} \right)$$

$$\boxed{h = \underline{\hspace{2cm}} \text{ mm Hg}}$$

To calculate the head in inches of water, we need to follow a similar procedure. The difference is that we need to use the density of water instead of the density of mercury, and use a conversion factor to convert meters to inches. Thus,

$$h = \frac{140 \text{ atm} \left(\frac{\underline{\hspace{2cm}} \frac{\text{N}}{\text{m}^2}}{1 \text{ atm}} \right) \left(\frac{\underline{\hspace{2cm}} \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{1 \text{ N}} \right)}{\left(\underline{\hspace{2cm}} \frac{\text{kg}}{\text{m}^3} \right) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}$$

$$h = 1446 \text{ m H}_2\text{O}$$

The density of water was obtained from Table A.2-3 of Geankoplis at a temperature of 25 °C. Converting this value to inches we get:

$$h = 1446 \text{ m H}_2\text{O} \left(\frac{1 \text{ in H}_2\text{O}}{\underline{\hspace{2cm}} \text{ m H}_2\text{O}} \right)$$

$$\boxed{h = \underline{\hspace{2cm}} \text{ in H}_2\text{O}}$$

Example 2.3-1: Diffusivity of Hydrogen inside a Fuel Cell

Hydrogen in a bipolar plate is diffusing in the anode side of a fuel cell. The current density of a stack of 440 fuel cells is $600 \frac{\text{mA}}{\text{cm}^2}$.

The hydrogen is entering the fuel cell at a pressure of 2 atm and a temperature of 25 °C. Determine the diffusivity of hydrogen through the gas-diffusion layer with a thickness of 100 μm in $\frac{\text{mm}^2}{\text{s}}$ if the fuel cell performance is limited by mass transfer.

The amount of hydrogen reacted as a function of current is described by the following equation:

$$\dot{n}_{\text{H}_2, \text{reacted}} = \frac{IN}{2F}$$

where:

I = current in amperes (A)

N = number of cells in the fuel cell stack

$$F = \text{Faraday's constant} = 96485 \frac{\text{C}}{\text{mol} \cdot \text{e}^-}$$

Strategy

The diffusivity of hydrogen can be determined using Fick's Law of diffusion and the definition of the consumption rate of hydrogen in terms of the current.

Solution

Fick's first law relates the diffusive flux to the difference in concentration of a substance. For this problem, Fick's first law is given by:

$$\dot{n}_{\text{H}_2, \text{reacted}} = -D_{\text{H}_2} A \frac{dC_{\text{H}_2}}{dx}$$

where:

D_{H_2} = diffusivity of hydrogen through the gas – diffusion layer

A = cross – sectional area of the gas – diffusion layer

$\frac{dC_{\text{H}_2}}{dx}$ = change in the concentration of hydrogen along the thickness of the gas – diffusion layer

The left hand side of Fick's first law can be re-written in terms of the current. Hence,

$$\frac{I}{A} = -D_{H_2} \frac{dC_{H_2}}{dx}$$

The problem statement is giving the value of the current density defined as the current of the fuel cell divided by the area. Thus, this equation can be rewritten in terms of the current density $\hat{I}$ as follows:

$$\hat{I} = -D_{H_2} \frac{dC_{H_2}}{dx}$$

Assuming the diffusivity remains constant along the gas-diffusion layer, this equation can be separated and integrated as follows:

$$\int_{C_{H_2@x=0}}^{C_{H_2@x=100\mu m}} \frac{dC_{H_2}}{C_{H_2}} = -\frac{\hat{I}}{D_{H_2}} \int_0^{100\mu m} dx$$

Since all the hydrogen entering the fuel cell is reacting when mass transfer is limiting the fuel cell performance, the concentration of hydrogen in the anode will be given by:

$$C_{H_2@x=100\mu m} = 0$$

The concentration of a substance is defined as the number of moles of substance in a volume of solution. Thus,

$$C_{H_2} = \frac{n_{H_2}}{V}$$

The concentration of hydrogen entering the channels in the bipolar plate can be obtained using ideal gas law as shown in the following steps:

$$PV = nRT$$

Solving for the concentration $\frac{n_{H_2}}{V}$ from the ideal gas law, we have:

$$C_{H_2} = \frac{n_{H_2}}{V} = \frac{P}{RT}$$

Substituting the corresponding quantities in the right side of this equation yields:

$$C_{H_2} = \frac{2 \text{ atm} \left(\frac{101325 \text{ Pa}}{1 \text{ atm}} \right)}{\left(\frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}} \right) (298.15 \text{ K})}$$

$$C_{H_2} = \frac{\text{mol}}{\text{m}^3}$$

Substituting the concentration values into the integral equation we get:

$$\int_{0 \text{ m}}^{1 \times 10^{-4} \text{ m}} dx = -D_{H_2} \int_{\frac{\text{mol}}{\text{m}^3}}^0 dC_{H_2}$$

We can integrate this equation and substitute the values for the current density, number of fuel cells and Faraday's constant to give:

$$\frac{\left(\frac{\text{mA}}{\text{cm}^2} \right) (\text{cells})}{2 \left(\frac{\text{C}}{\text{mol} \cdot e^-} \right)} [(1 \times 10^{-4}) \text{ m} - 0 \text{ m}] = -D_{H_2} \left(\frac{\text{mol}}{\text{m}^3} \right)$$

Finally we can solve for the diffusivity D_{H_2} to obtain:

$$D_{H_2} = \frac{\left(\frac{\text{mA}}{\text{cm}^2} \right) (\text{cells}) \left(\frac{1 \text{ A}}{1000 \text{ mA}} \right) \left(\frac{\text{C/s}}{1 \text{ A}} \right) \left(\frac{\text{cm}^2}{1 \text{ m}^2} \right)}{2 \left(\frac{\text{C}}{\text{mol} \cdot e^-} \right) \left(\frac{\text{mol}}{\text{m}^3} \right)} [(1 \times 10^{-4}) \text{ m}]$$

$$D_{H_2} = 1.67 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$$

This diffusivity value can be converted to $\frac{\text{mm}^2}{\text{s}}$ by multiplying it by a conversion factor from m^2 to mm^2 .

$$D_{H_2} = 1.67 \times 10^{-5} \frac{\text{m}^2}{\text{s}} \left(\frac{\text{mm}^2}{\text{m}^2} \right)$$

$$\boxed{D_{H_2} = \frac{\text{mm}^2}{\text{s}}}$$

Example 2.5-1: Reynolds Number of Hydrogen flowing into a Fuel Cell

A compressed gas tank contains hydrogen at room temperature and a pressure of 140 atm. The valve on this tank is opened and the hydrogen enters a fuel cell at a rate of $0.75 \frac{\text{kg}}{\text{hr}}$ through a steel pipe with an inner diameter of 5.46 mm. Determine the type of pipe connecting the fuel cell to the hydrogen tank and determine if the flow of hydrogen is laminar or turbulent.

Strategy

To determine the type of pipe used to connect the hydrogen tank to the fuel cell we need to find the pipe corresponding to the inner diameter given in the problem statement. The flow regime may be determined depending on the value of the Reynolds number.

Solution

Appendix A.5 of Geankoplis is showing the properties of different types of standard steel pipes. For the inner diameter of 5.46 mm, the nominal size of the pipe is $\frac{1}{8}$ in. with a Schedule Number of 80.

For flow inside a pipe, the Reynolds number is given by:

$$\text{Re} = \frac{Dv\rho}{\mu}$$

where:

D = inner diameter of the pipe in meters (m)

v = velocity of the fluid inside the pipe in $\frac{\text{m}}{\text{s}}$

ρ = density of fluid in $\frac{\text{kg}}{\text{m}^3}$

μ = viscosity of fluid in $\frac{\text{kg}}{\text{m} \cdot \text{s}}$

The velocity of hydrogen circulating in the pipes can be obtained by dividing the volumetric flow rate of hydrogen by the inner cross-sectional area of the pipe. However, since we are given the mass flow rate, we will have to calculate the volumetric flow rate using the ideal gas law.

$$P\dot{V} = \frac{\dot{m}RT}{M}$$

Solving for the volumetric flow rate $\dot{V}$ yields:

$$\dot{V} = \frac{\dot{m}RT}{MP}$$

Substituting the corresponding quantities into this equation we get:

$$\dot{V} = \frac{\left(0.75 \frac{\text{kg}}{\text{hr}}\right) \left(\frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}}\right) (298 \text{ K})}{(140 \text{ atm}) \left(\frac{101325 \text{ Pa}}{1 \text{ atm}}\right) \left(\frac{\text{kg}}{\text{kmol}}\right) \left(\frac{1 \text{ kmol}}{1000 \text{ moles}}\right)}$$

Note that conversion factors for the pressure and number of moles were used in order to get the correct units for the volumetric flow rate.

$$\dot{V} = \frac{\text{m}^3}{\text{hr}}$$

To calculate the velocity of hydrogen in the pipe, we also need to determine the cross-sectional area of the pipe. This value can be obtained directly from Table A.5 of Geankoplis.

$$A = \text{m}^2$$

Now we can obtain the velocity of hydrogen as shown in the following equations. Note that we are multiplying the velocity equation by the conversion factor from hours to seconds, hence to obtain the velocity in $\frac{\text{m}}{\text{s}}$.

$$v = \frac{\dot{V}}{A} = \frac{\frac{\text{m}^3}{\text{hr}} \left(\frac{1 \text{ hr}}{\text{s}}\right)}{\text{m}^2}$$

$$v = 0.772 \frac{\text{m}}{\text{s}}$$

The density of hydrogen at the pressure of 140 atm and the temperature of 25 °C (298 K) can be calculated using the ideal gas equation of state:

$$P\dot{V} = \frac{\dot{m}RT}{M}$$

Since the density ρ is equal to dividing the mass flow rate by the volumetric flow rate, we can solve for the density as shown in the following step.

$$\rho = \frac{\dot{m}}{\dot{V}} = \frac{\text{kg}}{\text{m}^3}$$

We can enter the numeric values into the right side of this equation to get:

$$\rho = \frac{\left(\text{_____ atm} \right) \left(\text{_____} \right) \left(\frac{2 \text{ kg}}{1 \text{ kmol}} \right) \left(\frac{1 \text{ kmol}}{1000 \text{ moles}} \right)}{\left(8.314 \frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}} \right) (298 \text{ K})} = \text{_____} \frac{\text{kg}}{\text{m}^3}$$

The only parameter left to be determined before calculating Reynolds number is the viscosity, which can be obtained from Figure A.3-2 for gases at a pressure of 1 atm. However, for hydrogen at the pressure and temperature conditions in this problem, the viscosity does not depend on pressure. Thus, it is valid to use Figure A.3-2.

To obtain the viscosity of hydrogen we need to locate the coordinates for hydrogen in Table A.3-8 and draw a line that passes through these coordinates and the temperature value of 25 °C. Hence, the viscosity will be estimated to be:

$$\mu \approx \text{_____} \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

Now we can enter the quantities we found for velocity, density and viscosity into the equation for the Reynolds number to yield:

$$\text{Re} = \frac{(5.46 \times 10^{-3} \text{ m}) \left(0.772 \frac{\text{m}}{\text{s}} \right) \left(\text{_____} \frac{\text{kg}}{\text{m}^3} \right)}{\text{_____} \frac{\text{kg}}{\text{m} \cdot \text{s}}}$$

$$\text{Re} = \text{_____}$$

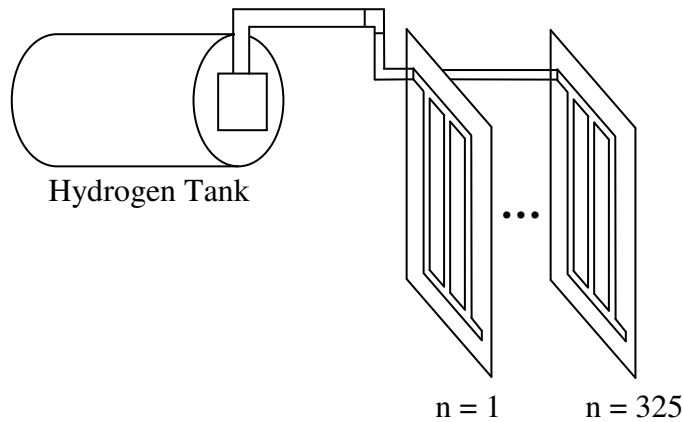
Conclusion:

Example 2.6-1: Flow of Hydrogen into Fuel Cells

Hydrogen is exiting the fuel tank in a vehicle through a stainless steel pipe (schedule number 80) with a nominal diameter of $\frac{1}{4}$ ". The hydrogen is leaving at a flow rate of $446.4 \frac{\text{L}}{\text{hr}}$ at room temperature and at a pressure of 2 atm. This flow rate is equally distributed between the 325 fuel cells required to power this vehicle.

The hydrogen is being distributed to the fuel cells by a pipe of $\frac{1}{16}$ " inner diameter. The hydrogen flow rate entering the channels of the bipolar plate of a fuel cell is equally distributed between the 3 channels of the bipolar plate. (*Note:* The channels in the bipolar plate will be assumed to be semicircular with an inner diameter of $\frac{1}{16}$ "

a) Determine the mass flow rate of hydrogen entering each cell and each one of the channels in the bipolar plate.



Strategy

To determine the mass flow rate from the volumetric flow rate we need to use ideal gas law.

Solution

We can solve the ideal gas law for the mass flow rate as shown in the following steps:

$$\dot{m}_{\text{overall}} = \frac{P \dot{V}_{\text{overall}} M}{RT}$$

Substituting the corresponding quantities into the right hand side of this equation yields:

$$\dot{m}_{\text{overall}} = \frac{(2 \text{ atm}) \left(446.4 \frac{\text{L}}{\text{hr}} \right) \left(2 \frac{\text{g}}{\text{mol}} \right)}{\left(\frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \right) (298.15 \text{ K})}$$

$$\dot{m}_{\text{overall}} = \frac{\text{g}}{\text{hr}}$$

To determine the flow rate of hydrogen entering each fuel cell, we divide the mass flow rate of hydrogen leaving the tank by the total number of fuel cells in the stack. Thus,

$$\dot{m}_{\text{fuel cell}} = \frac{\dot{m}}{n_{\text{fuel cells}}} = \frac{\frac{\text{g}}{\text{hr}}}{\text{cells}}$$

$$\dot{m}_{\text{fuel cell}} = 0.225 \frac{\text{g}}{\text{hr}}$$

This value can be divided by 3 (number of channels on each bipolar plate) to determine the flow of hydrogen to each channel:

$$\dot{m}_{\text{channel}} = \frac{\dot{m}_{\text{fuel cell}}}{n_{\text{channels}}} = \frac{0.225 \frac{\text{g}}{\text{hr}}}{3 \text{ channels}}$$

$$\dot{m}_{\text{channel}} = \frac{\text{g}}{\text{hr}}$$

b) What is the average velocity of the hydrogen leaving the tank?

Strategy

The velocity of hydrogen can be determined by applying the definition of velocity in terms of the flow rate.

Solution

The velocity of a fluid can be determined with the following equation:

$$v = \frac{\dot{V}}{A}$$

First we need to determine the volumetric flow rate of hydrogen to each fuel cell and to each channel in the bipolar plates. This can be done in a similar way to part a) of this problem.

$$\dot{V}_{\text{fuel cell}} = \frac{\dot{V}}{n_{\text{fuel cells}}} = \frac{446.4 \frac{\text{L}}{\text{hr}}}{325 \text{ cells}}$$

$$\dot{V}_{\text{fuel cell}} = \frac{\text{L}}{\text{hr}}$$

$$\dot{V}_{\text{channel}} = \frac{\dot{V}_{\text{fuel cell}}}{n_{\text{channels}}} = \frac{\text{L}}{3 \text{ channels hr}}$$

$$\dot{V}_{\text{channel}} = \frac{\text{L}}{\text{hr}}$$

Now we need to calculate the cross-sectional areas of different pipes through which hydrogen is circulating. First, the area of the 1/4" pipe connected to the fuel tank can be found in Table A.5-1 of Geankoplis to be:

$$A_{1/4" \text{ pipe}} = \text{m}^2$$

The cross-sectional area of the pipe entering the fuel cells can be calculated using the equation for the area of a circle, where the diameter will be 1/16". Hence,

$$A_{1/16" \text{ pipe}} = \left(\frac{\pi D^2}{4} \right) = \frac{\pi \left(\frac{1}{16} \text{ in} \right)^2}{4}$$

$$A_{1/16" \text{ pipe}} = \text{in}^2$$

Converting this value to m², we have:

$$A_{1/16" \text{ pipe}} = \text{in}^2 \left(\frac{\text{m}^2}{1 \text{ in}^2} \right)$$

$$A_{1/16" \text{ pipe}} = \text{m}^2$$

Since the channels on the bipolar plate are assumed to be semicircular, the cross-sectional area of the channel can be calculated as follows:

$$A_{\text{channel}} = \frac{\left(\frac{\pi D^2}{4} \right)}{2}$$

Substituting the corresponding quantities into the right hand side of this equation yields:

$$A_{\text{channel}} = \frac{\pi \left(\frac{1}{16} \text{ in} \right)^2 \left(\frac{\text{m}^2}{1 \text{ in}^2} \right)}{8}$$

$$A_{\text{channel}} = \text{_____ m}^2$$

Now we can obtain the velocities in the three different sections, as shown in the following equations:

$$v_{\text{H}_2 \text{ in } 1/4" \text{ pipe}} = \frac{\left(446.4 \frac{\text{L}}{\text{hr}} \right) \left(\frac{1 \text{ m}^3}{1000 \text{ L}} \right) \left(\frac{\text{_____}}{\text{_____}} \right)}{\text{_____ m}^2}$$

$$v_{\text{H}_2 \text{ in } 1/4" \text{ pipe}} = 2.68 \frac{\text{m}}{\text{s}}$$

$$v_{\text{H}_2 \text{ in fuel cell}} = \frac{\left(\frac{\text{_____ L}}{\text{hr}} \right) \left(\frac{1 \text{ m}^3}{1000 \text{ L}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right)}{\text{_____ m}^2}$$

$$v_{\text{H}_2 \text{ in fuel cell}} = \text{_____} \frac{\text{m}}{\text{s}}$$

$$v_{\text{H}_2 \text{ in channel}} = \frac{\left(\frac{\text{_____ L}}{\text{hr}} \right) \left(\frac{1 \text{ m}^3}{1000 \text{ L}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right)}{\text{_____ m}^2}$$

$$v_{\text{H}_2 \text{ in channel}} = \text{_____} \frac{\text{m}}{\text{s}}$$

c) Calculate the mass flux of hydrogen circulating through each channel in $\frac{\text{kg}}{\text{m}^2 \cdot \text{hr}}$.

Strategy

The flux of hydrogen refers to the amount of hydrogen flowing through an area during a period of time.

Solution

The flux of hydrogen can be obtained by dividing the mass flow rate of hydrogen entering each channel, by the cross sectional area of the channel. Thus,

$$G = \frac{\dot{m}_{\text{channel}}}{A_{\text{channel}}}$$

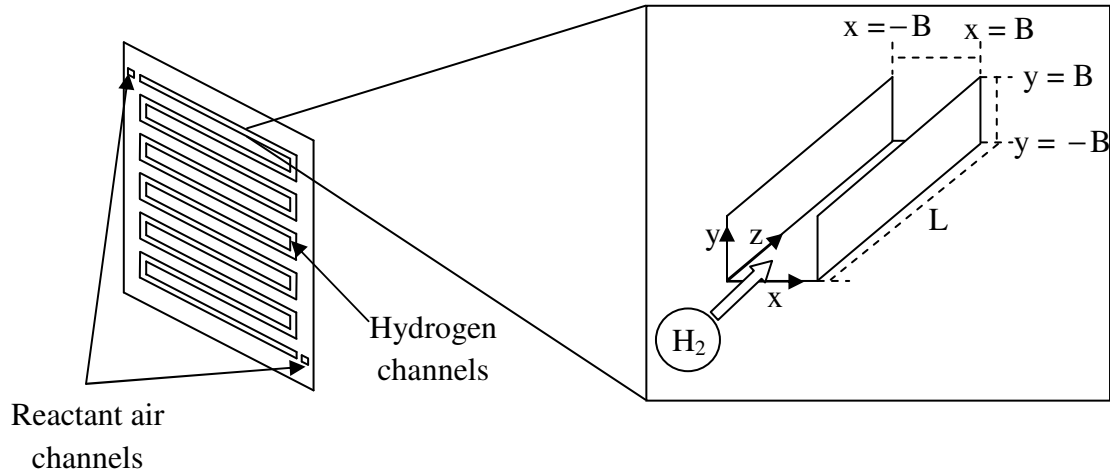
Entering numeric values into this equation we get:

$$G = \frac{\text{_____} \frac{\text{g}}{\text{hr}} \left(\frac{\text{_____}}{\text{_____}} \right)}{\text{_____} \text{ m}^2}$$

$G = \text{_____} \frac{\text{kg}}{\text{m}^2 \cdot \text{hr}}$

Example 2.6-3: Velocity of Hydrogen in Bipolar Plate Channel

The following figure is showing the flow of hydrogen along the horizontal channel of length L in a bipolar plate.



The velocity of hydrogen along a square channel in a bipolar plate is given by the following equation:

$$v_z = v_{\max} \left[1 - \left(\frac{x}{B} \right)^2 \right] \left[1 - \left(\frac{y}{B} \right)^2 \right]$$

Determine the average velocity of the hydrogen flowing through the channel.

Strategy

The average velocity can be calculated from the expression of the velocity as a function of the position in the x and y dimensions.

Solution

The average velocity can be calculated using Equation 2.6-17 of Geankoplis, shown below:

$$v_{\text{av}} = \frac{1}{A} \int_A v_z dA$$

In Cartesian coordinates dA may be written as $dx dy$. The cross-sectional area of the channel is obtained by multiplying the dimensions of the channel. Hence,

$$v_{\text{av}} = \frac{1}{(2B)^2} \int_{-B}^B \int_{-B}^B v_{\max} \left[1 - \left(\frac{x}{B} \right)^2 \right] \left[1 - \left(\frac{y}{B} \right)^2 \right] dx dy$$

Integrating this equation in the x-direction, we get:

$$v_{av} = \frac{v_{max}}{4B^2} \int_{-B}^B \left[x - \left(\frac{y^2}{B} \right) \right] \left[1 - \left(\frac{y}{B} \right)^2 \right] dy$$

Evaluating the integrated expression for x yields:

$$v_{av} = \frac{v_{max}}{4B^2} \int_{-B}^B \left\{ B - \left(\frac{y^2}{B} \right) - \left[-\frac{y^3}{3} - \left(\frac{y^5}{5B} \right) \right] \right\} \left[1 - \left(\frac{y}{B} \right)^2 \right] dy$$

We can reduce this equation to get:

$$v_{av} = \frac{v_{max}}{4B^2} \int_{-B}^B \left[\frac{y^3}{3} - \left(\frac{y^5}{5B} \right) \right] \left[1 - \left(\frac{y}{B} \right)^2 \right] dy$$

$$v_{av} = \frac{v_{max}}{4B^2} \int_{-B}^B \left(\frac{y^3}{3} - \frac{y^5}{5B} \right) \left[1 - \left(\frac{y}{B} \right)^2 \right] dy$$

$$v_{av} = \frac{v_{max}}{4B^2} \int_{-B}^B \left[1 - \left(\frac{y}{B} \right)^2 \right] dy$$

Now we can proceed to integrate and evaluate the equation for the average velocity with respect to y as shown in the following steps:

$$v_{av} = \frac{v_{max}}{4B^2} \left[\frac{y}{1} - \left(\frac{y^3}{3B} \right) \right]_{-B}^B$$

$$v_{av} = \frac{v_{max}}{4B^2} \left\{ \left[\frac{y}{1} - \left(\frac{y^3}{3B} \right) \right] - \left[-\frac{y}{1} - \left(\frac{y^3}{3B} \right) \right] \right\}$$

Reducing this equation we determine the expression for the average velocity of hydrogen in the channels:

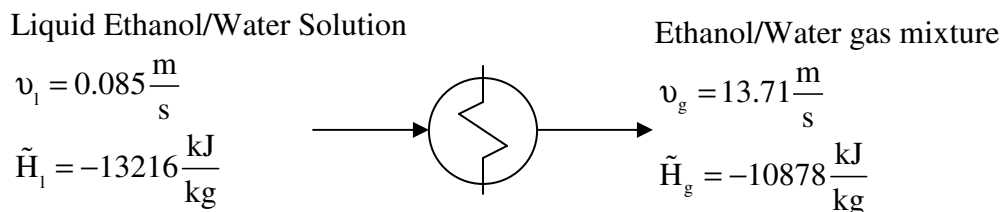
$$v_{av} = \frac{v_{max}}{4B^2} \left[\frac{2B^3}{3} \right]$$

$$v_{av} = \frac{v_{max}}{4B^2} \left(\frac{2B^3}{3} \right) = \frac{v_{max}}{6}$$

$$\boxed{v_{av} = \frac{4v_{max}}{9}}$$

Example 2.7-1: Energy Balance on Ethanol Boiler

A liquid solution of ethanol and water is entering a boiler before undergoing a reforming reaction for producing hydrogen for proton-exchange membrane fuel cells. A diagram of the process is shown below.



What is the power required to produce $1323 \frac{\text{kg}}{\text{hr}}$ of vapor?

Strategy

The overall energy balance for a steady-state flow system can be used to determine the amount of heat required.

Solution

The energy balance equation is given by Equation 2.7-10 of Geankoplis and is shown below:

$$H_2 - H_1 + \frac{1}{2\alpha}(v_2^2 - v_1^2) + g(z_2 - z_1) = Q - W_s$$

where:

H_2 = Enthalpy of the substance exiting the system

H_1 = Enthalpy of the substance entering the system

α = Kinetic-energy velocity correction factor

v_2 = Velocity of the substance exiting the system

v_1 = Velocity of the substance entering the system

g = Acceleration due to gravity

$z_2 - z_1$ = Difference in height between the outlet and inlet points in the system

Q = Heat added (+) or removed (–) to the system

W_s = External work applied by (+) or to (–) the system

For this problem we will assume turbulent flow, so the value of the kinetic-energy velocity correction factor α is close to 1. The external work term W_s will be neglected in the balance equations as there are no mechanical parts performing work to the system. Since no information is given about the height of the inlet and outlet points in the system, we will assume the values are the same. Thus, the difference $z_2 - z_1$ is equal to zero.

After applying these assumptions, the energy balance equation will be given by:

$$\tilde{Q} = \tilde{H}_2 - \tilde{H}_1 + \frac{1}{2}(\text{_____} - \text{_____})$$

Where the tilde represents the amount per unit mass.

The values given for the mass enthalpies of the ethanol/water mixtures and the velocities of the liquid and gas can be now substituted into this equation to obtain the amount of heat required. Thus,

$$\tilde{Q} = \tilde{H}_2 - \tilde{H}_1 + \frac{1}{2}(\text{_____} - \text{_____})$$

$$\tilde{Q} = \text{_____} \frac{\text{kJ}}{\text{kg}} - \left(-13216 \frac{\text{kJ}}{\text{kg}}\right) + \frac{1}{2} \left[\left(13.71 \frac{\text{m}}{\text{s}}\right)^2 - \left(\text{_____} \frac{\text{m}}{\text{s}}\right)^2 \right]$$

$$\tilde{Q} = 2432 \frac{\text{kJ}}{\text{kg}}$$

To determine the power required, we need to multiply the heat per kilogram of vapor by the flow rate:

$$\dot{Q} = 2432 \frac{\text{kJ}}{\text{kg}} \left(\text{_____} \frac{\text{kg}}{\text{hr}} \right) \left(\frac{\text{_____}}{\text{_____}} \right)$$

$$\boxed{\dot{Q} = \text{_____} \text{ kW}}$$

Example 2.10-1: Methanol Flow in Fuel Cell

A solution of 40 wt. % methanol and 60 wt. % water is entering the channels of a bipolar plate in a direct methanol fuel cell. The viscosity and density of this solution are shown below:

$$\mu = 1.85 \times 10^{-3} \text{ Pa} \cdot \text{s}$$

$$\rho = 931.5 \frac{\text{kg}}{\text{m}^3}$$

The pressure drop along a rectangular channel is shown by Bahrami et al. [1] to be:

$$\Delta P = \frac{\mu L v}{c^2 \left[\frac{1}{3} - \frac{64\epsilon}{\pi^5} \tanh\left(\frac{\pi}{2\epsilon}\right) \right]}$$

where:

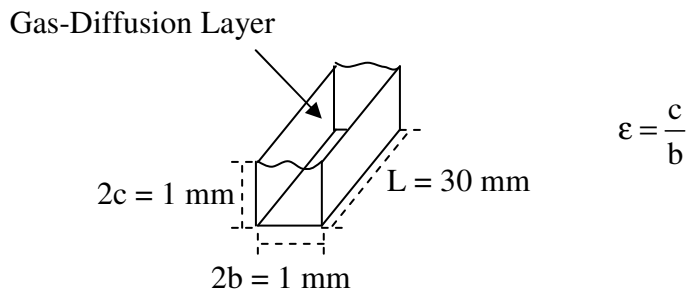
ΔP = Pressure drop, Pa

μ = Viscosity of the fluid, Pa · s

L = Length of the channel, m

v = Velocity of the fluid in the channel, $\frac{\text{m}}{\text{s}}$

The parameters c and ϵ are related to the dimensions of the channel as shown in the following figure:



What is the pressure drop in atm and mm Hg along a single channel if the methanol flowing in the fuel cell has a Reynolds number of 300?

Strategy

The pressure drop in the channel can be calculated using the equation for the pressure drop given in the problem statement.

1. Bahrami, M., Yovanovich, M. M., Culham, J. R., "Pressure Drop of Fully-Developed, Laminar Flow in Microchannels of Arbitrary Cross Section, *Journal of Fluids Engineering*, **128**, 1036-1044 (2006)

Solution

The equation given in the problem statement for the pressure drop depends on the velocity of the fluid in the channel. Since we are not given this value directly, we will have to calculate it from the definition of the Reynolds number. For an open channel, the Reynolds number is defined as follows:

$$Re = \frac{D_H v \rho}{\mu}$$

where D_H is the hydraulic diameter, given by:

$$D_H = \frac{4A}{P}$$

In this equation, A is the cross-sectional area of the channel, and P is the perimeter of the channel in contact with the fluid. Since the fluid in the channel is in contact with the gas diffusion layer, we need to consider all the dimensions of the channel when calculating the wetted perimeter. Hence, the hydraulic diameter can be calculated using the dimensions of the channel as shown in the following step:

$$D_H = \frac{4(2b \cdot 2c)}{\quad}$$

Substituting the values of b and c into this equation yields:

$$D_H = \frac{4(1 \text{ mm})^2}{\quad} \left(\frac{1 \text{ m}}{1000 \text{ mm}} \right)$$

$$D_H = 1 \times 10^{-3} \text{ m}$$

Now we can enter this value for the hydraulic diameter into the equation for Reynolds number and solve for the velocity to get:

$$v = \frac{Re \mu}{D_H \rho} = \frac{300 \left(1.85 \times 10^{-3} \frac{\text{kg}}{\text{m} \cdot \text{s}} \right)}{(1 \times 10^{-3} \text{ m}) \left(931.5 \frac{\text{kg}}{\text{m}^3} \right)}$$

$$v = \frac{\quad}{\quad} \frac{\text{m}}{\text{s}}$$

The only remaining values that need to be calculated before being able to determine the pressure drop are the parameters b , c and ε :

$$2b = 2c = \underline{\hspace{2cm}}$$

$$b = c = \frac{\underline{\hspace{2cm}}}{2} \left(\frac{1 \text{ m}}{1000 \text{ mm}} \right) = \underline{\hspace{2cm}} \text{ m}$$

$$\epsilon = \frac{c}{b} = \frac{\underline{\hspace{2cm}} \text{ m}}{5 \times 10^{-4} \text{ m}}$$

$$\epsilon = \underline{\hspace{2cm}}$$

Finally, the pressure drop along the length of the channel of 30 mm can be calculated as follows:

$$\Delta P = \frac{\left(1.85 \times 10^{-3} \frac{\text{kg}}{\text{m} \cdot \text{s}} \right) (0.03 \text{ m}) \left(\underline{\hspace{2cm}} \frac{\text{m}}{\text{s}} \right) \left(\frac{1 \text{ atm}}{101325 \text{ Pa}} \right)}{\left(\underline{\hspace{2cm}} \right)^2 \left[\frac{1}{3} - \frac{64(1)}{\pi^5} \tanh \left(\frac{\pi}{2(1)} \right) \right]}$$

$$\Delta P = \underline{\hspace{2cm}} \text{ atm}$$

Converting this value to mm Hg, we have:

$$\Delta P = \underline{\hspace{2cm}} \text{ atm} \left(\frac{\underline{\hspace{2cm}}}{\underline{\hspace{2cm}}} \right)$$

$$\Delta P = \underline{\hspace{2cm}} \text{ mm Hg}$$

Example 2.10-2: Use of Friction Factor in Laminar Flow

An aqueous solution of 40 mol % methanol is flowing along the channels in a bipolar plate of a direct-methanol fuel cell at a velocity of $0.601 \frac{\text{m}}{\text{s}}$ and a temperature of 27°C .

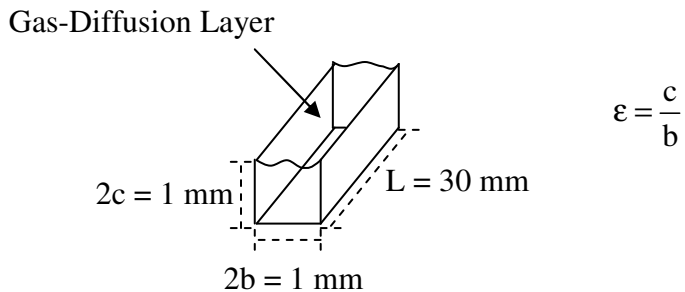
Bahrami et al.[1] are showing the following equation to determine the friction factor in a rectangular channel:

$$f \cdot \text{Re}_{\sqrt{A}} = \frac{12}{\left[1 - \frac{192\varepsilon}{\pi^5} \tanh\left(\frac{\pi}{2\varepsilon}\right) \right] (1 + \varepsilon) \sqrt{\varepsilon}}$$

where:

$$\text{Re}_{\sqrt{A}} = \frac{v \rho \sqrt{A}}{\mu}$$

The parameter ε is a function of the dimensions of the channel, as shown in the following figure.



Use the definition of friction factor to calculate the pressure drop in atm along a single channel.

Strategy

The pressure drop in the channel can be calculated using the equation for the friction factor given in Geankoplis.

Solution

Equation 2.10-4 of Geankoplis gives the definition of Fanning friction factor:

$$f = \frac{\left(\frac{A \Delta P}{A_w} \right)}{\frac{\rho v^2}{2}}$$

1. Bahrami, M., Yovanovich, M. M., Culham, J. R., "Pressure Drop of Fully-Developed, Laminar Flow in Microchannels of Arbitrary Cross Section, *Journal of Fluids Engineering*, **128**, 1036-1044 (2006)

where:

A = Cross – sectional area of the channel.

A_w = Surface area of the channel being wetted by the fluid.

ΔP = Pressure – drop along the channel of length L

ρ = Density of the fluid circulating through the channel

v = Velocity of the fluid in the channel

We can start by solving for the pressure drop ΔP from the definition of friction factor, given in Equation 2.10-4 of Geankoplis.

$$\Delta P = \frac{\text{_____}}{\text{_____}} \left(\frac{\text{_____}}{2} \right)$$

For the rectangular channel in the bipolar plate, the cross sectional area and the wetted surface area are given by:

$$A = (2b)(2c)$$

$$A_w = [2(\text{_____})]L$$

Note that for the wetted perimeter we are considering the area of the channel in contact with the gas-diffusion layer.

Since $b = c$, we can rewrite the area and the wetted perimeter as follows and substitute the dimensions of the channel in these equations to get:

$$A = \text{_____} = 4(\text{_____ mm})^2 \left(\frac{1 \text{ m}^2}{\text{_____ mm}^2} \right) = \text{_____ m}^2$$

$$A_w = \text{_____} = 8(\text{_____ mm})(\text{_____ mm}) \left(\frac{1 \text{ m}^2}{1 \times 10^6 \text{ mm}^2} \right) = \text{_____ m}^2$$

the value of ϵ and enter the obtained value into the equation for $f \cdot \text{Re}_{\sqrt{A}}$. Thus,

$$\epsilon = \frac{c}{b} = \frac{0.5 \text{ mm}}{\text{_____ mm}} = \text{_____}$$

$$f \cdot \text{Re}_{\sqrt{A}} = \frac{12}{\left[1 - \frac{192}{\pi^5} \tanh\left(\frac{\pi}{2}\right)\right] \left(1 + \frac{\text{Re}_{\sqrt{A}}}{1000}\right) \sqrt{\text{Re}_{\sqrt{A}}}}$$

$$f \cdot \text{Re}_{\sqrt{A}} = \frac{12}{\left[1 - \frac{192}{\pi^5} \tanh\left(\frac{\pi}{2}\right)\right] \left(1 + \frac{\text{Re}_{\sqrt{A}}}{1000}\right) \sqrt{\text{Re}_{\sqrt{A}}}}$$

The Fanning friction factor f can be obtained from this equation but first we need to calculate the Reynolds number $\text{Re}_{\sqrt{A}}$ using the equation given in the problem statement as shown in the following steps:

$$\text{Re}_{\sqrt{A}} = \frac{v \rho (\sqrt{A})}{\mu}$$

The density of methanol at the temperature of 27 °C can be obtained from Table 2-234 of *Perry's Chemical Engineers' Handbook*, 8th Edition to be:

$$\rho_{\text{CH}_3\text{OH}} = 24.486 \frac{\text{mol}}{\text{L}}$$

Converting this value to $\frac{\text{kg}}{\text{m}^3}$, we have:

$$\rho_{\text{CH}_3\text{OH}} = 24.486 \frac{\text{mol}}{\text{L}} \left(\frac{\text{g CH}_3\text{OH}}{1 \text{ mol CH}_3\text{OH}} \right) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) \left(\frac{\text{L}}{1 \text{ m}^3} \right)$$

$$\rho_{\text{CH}_3\text{OH}} = \frac{\text{kg}}{\text{m}^3}$$

Since the methanol entering the fuel cell is diluted in water, the density will depend on the concentration of the methanol solution. Thus, the density can be calculated as shown below:

$$\rho = \frac{V_{\text{CH}_3\text{OH}}}{V_{\text{total}}} \rho_{\text{CH}_3\text{OH}} + \frac{V_{\text{H}_2\text{O}}}{V_{\text{total}}} \rho_{\text{H}_2\text{O}}$$

The density of water is given in Table A.2-3 of Geankoplis. Since the value of the density at the temperature of 27 °C, we can use the value in the Table at the temperature closest to 27 °C, which is 25 °C:

$$\rho_{\text{H}_2\text{O}} = \frac{\text{kg}}{\text{m}^3}$$

Substituting the individual densities of water and methanol into the equation for the density of the solution ρ yields:

$$\rho = 0.4 \left(\frac{\text{kg}}{\text{m}^3} \right) + 0.6 \left(\frac{\text{kg}}{\text{m}^3} \right)$$

$$\rho = \frac{\text{kg}}{\text{m}^3}$$

The viscosity of methanol can be determined using Figure A.3-4 of Geankoplis and the coordinates in Table A.3-12. Thus,

$$\mu = \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

Now we can obtain the value of the Reynolds number $\text{Re}_{\sqrt{A}}$ as follows:

$$\text{Re}_{\sqrt{A}} = \frac{v \rho (\sqrt{A})}{\mu} = \frac{\left(0.601 \frac{\text{m}}{\text{s}} \right) \left(\frac{\text{kg}}{\text{m}^3} \right) \left(\sqrt{\text{m}^2} \right)}{\frac{\text{kg}}{\text{m} \cdot \text{s}}}$$

$$\text{Re}_{\sqrt{A}} =$$

From the value calculated for $f \cdot \text{Re}_{\sqrt{A}}$ we can solve for the Fanning friction factor to yield:

$$f = \frac{14.13}{\text{Re}_{\sqrt{A}}} = \frac{14.13}{}$$

$$f =$$

Finally we can determine the value of the pressure drop along the channel in the direct-methanol fuel cell by entering all the corresponding values into the equation for ΔP

$$\Delta P = \frac{(1.2 \times 10^{-4} \text{ m}^2)}{\text{m}^2} \cdot \frac{\frac{\text{kg}}{\text{m}^3} \left(0.601 \frac{\text{m}}{\text{s}} \right)^2}{2}$$

$$\Delta P = 942.45 \text{ Pa} \left(\frac{1 \text{ atm}}{\text{Pa}} \right)$$

$$\Delta P = \text{ atm}$$

Example 2.10-3: Use of Friction Factor in Turbulent Flow

Pure hydrogen is exiting a pressure swing adsorption unit at a rate of $63.12 \frac{\text{kg}}{\text{hr}}$ and a pressure of 2061 kPa. The hydrogen is entering the storage tank through a pipe with an inner diameter of 100 mm and a length of 10 m. If the maximum friction loss permitted along the pipe is $3.60 \frac{\text{J}}{\text{kg}}$, what material would you propose for the pipe transporting the hydrogen from the pressure swing adsorption unit to the storage tank?

The gas leaving the pressure swing adsorption unit has the following properties:

$$\mu = 9.769 \times 10^{-6} \text{ Pa} \cdot \text{s}$$

$$\rho = 1.425 \frac{\text{kg}}{\text{m}^3}$$

Strategy

To determine what type of pipe to use in this process, we need to determine the parameter ϵ for the flow conditions in this process.

Solution

The parameter $\frac{\epsilon}{D}$ may be obtained from Figure 2.10-3 of Geankoplis. To do this, we need to know the values of the Reynolds number and the Fanning friction factor.

The friction factor f can be calculated from the definition of the friction loss, as shown in the following equation:

$$F_f = 4f \frac{\Delta L}{D} \frac{v^2}{2}$$

Solving for the Fanning friction factor f , we get:

$$f = \frac{\text{_____}}{\text{_____}}$$

The value of the velocity of hydrogen inside the pipe is not given. However, its value can be calculated by dividing the volumetric flow rate of hydrogen by the cross-sectional area of the pipe. Hence,

$$v = \frac{\dot{V}}{A}$$

The volumetric flow rate is calculated by dividing the mass flow rate by the density. The calculation of the volumetric flow rate and the cross-sectional area of the pipe are shown in the following steps:

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.1 \text{ m})^2}{4} = \text{_____ m}^2$$

$$v = \frac{\dot{m}}{\rho A} = \frac{63.12 \frac{\text{kg}}{\text{hr}}}{\left(1.425 \frac{\text{kg}}{\text{m}^3}\right) \left(\text{_____ m}^2\right)} \left(\frac{1 \text{ hr}}{3600 \text{ s}}\right)$$

$$v = \text{_____} \frac{\text{m}}{\text{s}}$$

Substituting this velocity value into the equation for the friction factor we obtained, we have:

$$f = \text{_____}$$

The other parameter required to locate the parameter $\frac{\epsilon}{D}$ in Figure 2.10-3 is the Reynolds number, which is obtained as follows:

$$\text{Re} = \frac{D v \rho}{\mu}$$

Substituting the corresponding values into this equation yields:

$$\text{Re} = \frac{(0.1 \text{ m}) \left(\text{_____} \frac{\text{m}}{\text{s}}\right) \left(1.425 \frac{\text{kg}}{\text{m}^3}\right)}{9.769 \times 10^{-6} \frac{\text{kg}}{\text{m} \cdot \text{s}}}$$

$$\text{Re} = \text{_____}$$

Now we can locate the value of $\frac{\epsilon}{D}$ in the graph for the Reynolds number and the friction factor.

From Figure 2.10-3, this parameter is estimated to be:

$$\frac{\epsilon}{D} \approx \text{_____}$$

Solving for the pipe roughness ε , we get:

$$\varepsilon \approx \text{_____} D \approx \text{_____} (0.1 \text{ m})$$

$$\varepsilon \approx \text{_____} \text{ m}$$

Conclusion:

Example 2.10-4: Trial-and-Error Solution to Calculate Pipe Diameter

Hydrogen in a fuel cell vehicle is flowing from the tank to the fuel cell stack through a commercial steel pipe with a length of 3 m. The hydrogen is being consumed at a rate of $1.17 \frac{\text{g}}{\text{s}}$ and is entering the fuel cell stack at a temperature of 25 °C and a pressure of 2.5 atm.

Determine the diameter of the pipe connecting the fuel tank to the fuel cell stack if a head of 240 in Hg is available to compensate for the friction loss.

Strategy

From the value of F_f given as head of fluid we can determine the required diameter for feeding hydrogen to the fuel cells.

Solution

The friction loss can be calculated by multiplying the head of fluid in m of H_2O by the acceleration due to gravity. Thus,

$$F_f = 240 \text{ in Hg} \left(\frac{\text{m}}{\text{s}^2} \right) \left(\frac{\text{m H}_2\text{O}}{1 \text{ in Hg}} \right)$$

$$F_f = 812.69 \frac{\text{J}}{\text{kg}}$$

To determine the pipe diameter we have to obtain the diameter from the definition of friction force, given by the following equation:

$$F_f = 4f \frac{\Delta L}{D} \frac{v^2}{2}$$

Solving for the diameter D, we have:

$$D = 4f \frac{\Delta L}{F_f} \frac{v^2}{2}$$

As it can be seen in this equation, we need to obtain the Fanning friction factor f and the velocity v before being able to calculate the diameter.

The friction factor can be obtained from Figure 2.10-3 of Geankoplis. To do this, we need to calculate Reynolds number first and obtain the relative roughness $\frac{\epsilon}{D}$ of commercial steel. Hence,

$$\frac{\epsilon}{D} = \frac{\text{m}}{\text{m}}$$

Before obtaining the velocity of the fluid first we need to convert the mass flow rate to volumetric flow rate. To do this, we need to calculate the density of the fluid using the ideal gas equation of state. The velocity can then be determined by dividing the volumetric flow rate of hydrogen by the cross-sectional area of the pipe as shown in the following steps:

$$\rho = \frac{PM}{RT} = \frac{(2.5 \text{ atm}) \left(\frac{2 \text{ g}}{\text{mol}} \right) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right)}{\frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} (298.15 \text{ K}) \left(\frac{1 \text{ m}^3}{1000 \text{ L}} \right)} = \frac{\text{kg}}{\text{m}^3}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{1.17 \frac{\text{g}}{\text{s}} \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right)}{\frac{\text{kg}}{\text{m}^3}} = \frac{\text{m}^3}{\text{s}}$$

$$A = \frac{\pi D^2}{4}$$

$$v \left(\frac{\text{m}}{\text{s}} \right) = \frac{\dot{V}}{A} = \frac{4 \left(\frac{\text{m}^3}{\text{s}} \right)}{\pi D^2}$$

$$v = \frac{\dot{V}}{A}$$

The other parameter needed to obtain the friction factor from Figure 2.10-3 is the Reynolds number, calculated as follows:

$$\text{Re} = \frac{D v \rho}{\mu} = \frac{D (\text{m}) \left(\frac{\text{m}}{\text{s}} \right) \left(\frac{\text{kg}}{\text{m}^3} \right)}{\frac{\text{kg}}{\text{m} \cdot \text{s}}}$$

$$\text{Re} = \frac{169.23}{D}$$

The viscosity was obtained from Appendix A.3 of Geankoplis.

As we can see, the diameter of the pipe D appears in all the expressions required for determining the friction factor. Therefore, we will select a diameter value and compare the result obtained to the calculated friction force. The initial guess for the diameter will be:

Trial 1

$$D = 0.010 \text{ m}$$

Substituting this value in the equations for Reynolds number, velocity and relative roughness, we get:

$$Re = \frac{169.23}{D} = \frac{169.23}{0.010} = \underline{\hspace{2cm}}$$

$$v = \frac{\hspace{1cm}}{D^2} = \frac{\hspace{1cm}}{(0.010)^2} = \underline{\hspace{1cm}} \frac{\text{m}}{\text{s}}$$

$$\frac{\epsilon}{D} = \frac{\hspace{1cm} \text{m}}{D} = \frac{\hspace{1cm} \text{m}}{(0.010 \text{ m})} = \underline{\hspace{2cm}}$$

Locating the values of the relative roughness and the Reynolds number in Figure 2.10-3 we find:

$$f = 0.0085$$

Substituting this result as well as the velocity and dimensions of the pipe into the equation for the friction force we get:

$$F_f = 4f \frac{\Delta L}{D} \frac{v^2}{2}$$

$$F_f = 4(0.0085) \frac{\hspace{1cm} \text{m}}{0.010 \text{ m}} \frac{\left(\underline{\hspace{1cm}} \frac{\text{m}}{\text{s}} \right)^2}{2}$$

$$F_f = \underline{\hspace{2cm}} \frac{\text{J}}{\text{kg}}$$

It can be seen that this value does not match the calculated friction force. For a second trial, we will select a diameter of 0.020 m. Selecting a higher value for the diameter, will result in an decrease in the velocity of the fluid, thus reducing the value of the friction force.

Trial 2

$$D = 0.020 \text{ m}$$

Substituting this value in the equations for Reynolds number, velocity and relative roughness, we get:

$$Re = \frac{169.23}{D} = \frac{169.23}{0.020} = \underline{\hspace{2cm}}$$

$$v = \frac{\text{_____}}{D^2} = \frac{\text{_____}}{(0.020)^2} = \text{_____} \frac{\text{m}}{\text{s}}$$

$$\frac{\epsilon}{D} = \frac{\text{_____ m}}{D} = \frac{\text{_____ m}}{(0.020 \text{ m})} = \text{_____}$$

$$f = \text{_____}$$

$$F_f = 4 \left(\text{_____} \right) \frac{\text{m}}{0.020 \text{ m}} \frac{\left(\text{_____} \frac{\text{m}}{\text{s}} \right)^2}{2}$$

$$F_f = \text{_____} \frac{\text{J}}{\text{kg}}$$

As we can see, the value obtained for a diameter of 0.020 m yields a friction force of _____ $\frac{\text{J}}{\text{kg}}$. The error between this value and the friction force calculated using the head of fluid available is given by:

$$\text{Error \%} = \frac{\text{_____} \frac{\text{J}}{\text{kg}} - \text{_____} \frac{\text{J}}{\text{kg}}}{\text{_____} \frac{\text{J}}{\text{kg}}} (100\%)$$

$$\text{Error} = \text{_____} \%$$

Conclusion:

Example 2.10-5: Flow of Gas in Line and Pressure Drop

Hydrogen at room temperature and standard pressure is entering a fuel cell stack through a smooth pipe with an inner diameter of 1 cm and a length of 3 m. Calculate the pressure of hydrogen in the fuel tank. The hydrogen consumption rate is $2.053 \frac{\text{g}}{\text{s}}$.

Strategy

The initial pressure of the gas can be determined using the equation for the pressure drop of a gas inside a tube.

Solution

We will start by assuming turbulent flow of hydrogen in the pipe. The initial pressure of hydrogen can be calculated using Equation 2.10-10 of Geankoplis shown below:

$$P_1^2 - P_2^2 = 4f \frac{\Delta L \cdot G^2 R T}{D M}$$

where:

P_1 = Initial pressure of the fluid in the pipe, Pa

P_2 = Final pressure of the fluid in the pipe, Pa

f = Fanning friction factor

ΔL = Length of the tube, m

G = Mass flux of the fluid in the pipe, $\frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$

R = Ideal gas constant, $\frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}}$

T = Temperature of the fluid, K

D = Inside diameter of the pipe, m

M = Molecular weight of the fluid, $\frac{\text{g}}{\text{mol}}$

The flux of hydrogen may be calculated by dividing the mass flow rate of hydrogen by the cross-sectional area of the pipe as shown in the following steps.

$$G = \frac{\dot{m}}{A}$$

$$A = \frac{\pi D^2}{4} = \frac{\pi (\text{_____ m})^2}{4} = \text{_____ m}^2$$

Substituting this value and the mass flow rate into the equation for the mass flux of hydrogen we get:

$$G = \frac{2.053 \frac{\text{g}}{\text{s}} \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right)}{\text{_____ m}^2}$$

$$G = \text{_____} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$

The friction factor can be obtained from Figure 2.10-3 of Geankoplis. To do this, first we need to calculate Reynolds number and the relative roughness of the pipe $\frac{\epsilon}{D}$. Thus,

$$\text{Re} = \frac{DG}{\mu}$$

Substituting the corresponding values for the parameters on the right side of this equation yields:

$$\text{Re} = \frac{0.01 \text{ m} \left(\text{_____} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}} \right)}{\left(8.8 \times 10^{-6} \frac{\text{kg}}{\text{m} \cdot \text{s}} \right)} = \text{_____}$$

The viscosity of hydrogen was obtained from Appendix A.3 of Geankoplis. We can see that this value for the Reynolds number indicates that the flow of hydrogen to the fuel cell is turbulent. Therefore, the equation we selected for calculating the pressure P_1 is valid for this problem.

Now with the Reynolds number value we can obtain the friction factor for a smooth pipe to be given by:

$$f = \text{_____}$$

Solving Equation 2.10-10 of Geankoplis for the pressure P_1 and substituting all the known quantities into this equation, we can obtain the pressure of hydrogen leaving the fuel tank as shown in the following steps.

$$P_1 = \sqrt{4f \frac{\Delta L \cdot G^2 RT}{DM} + P_2^2}$$

$$P_1 = \sqrt{4 \left(\frac{3 \text{ m} \cdot \left(\frac{\text{kg}}{\text{m}^2 \cdot \text{s}} \right)^2 \left(\frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}} \right) (298.15 \text{ K})}{(0.01 \text{ m}) \left(2 \frac{\text{g}}{\text{mol}} \right) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right)} + \left[1 \text{ atm} \left(\frac{101325 \text{ Pa}}{1 \text{ atm}} \right) \right]^2}$$

$$P_1 = \frac{\text{Pa}}{101325 \text{ Pa}} \left(\frac{1 \text{ atm}}{101325 \text{ Pa}} \right)$$

$P_1 = \frac{\text{atm}}{101325 \text{ Pa}} \left(\frac{1 \text{ atm}}{101325 \text{ Pa}} \right)$

Example 2.10-8: Entry Length for a Fluid in a Rectangular Channel

Determine if the velocity profile for the gas flowing through the channels of a fuel cell bipolar plate is fully developed for the following cases:

a) Hydrogen in a Proton-Exchange Membrane Fuel Cell at a temperature of 25 °C with laminar flow. The fuel consumption rate is of $1.642 \frac{\text{g H}_2}{\text{s}}$.

b) Carbon monoxide in a Solid - Oxide Fuel Cell at a temperature of 800 °C with laminar flow. The fuel cell is converting carbon monoxide into products at a rate of $0.51 \frac{\text{kg CO}}{\text{hr}}$.

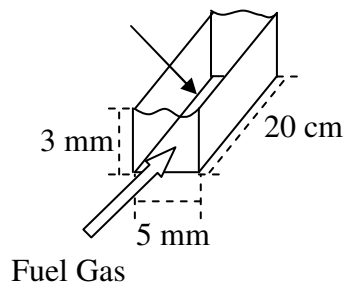
The viscosity of carbon monoxide was found in Appendix A.3-2 to be:

$$\mu = 3.8 \times 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

c) Same gases from parts a) and b) with turbulent flow.

The dimensions of the channels in the bipolar plate are shown in the following figure:

Gas-Diffusion Layer



Strategy

The entry length is the distance required for the establishment of fully-developed velocity profile.

Solution

a) To determine if the velocity profile is fully established in the channel of a bipolar plate, we need to calculate the entry length L_e , defined by the following equation:

$$\frac{L_e}{D} = 0.0575 \text{ Re}$$

For this problem, the diameter D of the channel will be equal to the hydraulic diameter, i.e. the perimeter of the channel 'wetted' by the fluid flowing through it. Thus,

$$\frac{L_e}{D_H} = 0.0575 \text{ Re} \quad (1)$$

To determine the entry length, we need to calculate the Reynolds number, as shown in the following steps:

$$\text{Re} = \frac{D_H G}{\mu}$$

The mass flux of hydrogen G is obtained by dividing the mass flow rate of hydrogen by the cross-sectional area of the channel. Hence,

$$G = \frac{\dot{m}}{A} = \frac{1.642 \frac{\text{g H}_2}{\text{s}} \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right)}{(\text{ } \text{mm})(\text{ } \text{mm}) \left(\frac{1 \text{ m}^2}{\text{ } \text{mm}^2} \right)} = \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$

The hydraulic diameter is calculated using the following equation

$$D_H = \frac{4A}{P} = \frac{4(\text{ } \text{m}^2)}{2(0.005 \text{ m} + 0.003 \text{ m})} = \text{ } \text{m}$$

where:

A = Cross-sectional area of the channel

P = Perimeter of the channel 'wetted' by the fluid.

Substituting this value and the mass flux in the equation for Reynolds number we get:

$$\text{Re} = \frac{(\text{ } \text{m}) \left(\frac{\text{kg}}{\text{m}^2 \cdot \text{s}} \right)}{\left(8.8 \times 10^{-6} \frac{\text{kg}}{\text{m} \cdot \text{s}} \right)} = \text{ }$$

The viscosity of hydrogen was obtained from Appendix A.3 of Geankoplis. Now we can solve equation (1) for the entry length L_e and substitute the calculated values to yield:

$$L_e = 0.0575 \text{ Re } D_H = 0.0575 (\text{ }) (\text{ } \text{m})$$

$$L_e = \text{ } \text{m}$$

Conclusion:

b) The entry length for the carbon monoxide flowing in the solid-oxide fuel cell will be determined in a similar way to part a) of this problem.

$$\frac{L_e}{D_H} = 0.0575 \text{ Re}$$

The calculations of the mass flux and Reynolds number of carbon monoxide are shown in the following steps:

$$G = \frac{\dot{m}}{A} = \frac{\frac{\text{kg CO}}{\text{hr}} \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right)}{(3 \text{ mm}) \left(\frac{\text{mm}}{\text{mm}} \right) \left(\frac{1 \text{ m}^2}{\text{mm}^2} \right)} = \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$

The hydraulic diameter from part a) is given by:

$$D_H = \text{_____ m}$$

Substituting the hydraulic and the mass flux of CO in the equation for Reynolds number yields:

$$\text{Re} = \frac{\left(\text{_____ m} \right) \left(\frac{\text{kg}}{\text{m}^2 \cdot \text{s}} \right)}{\frac{\text{kg}}{\text{m} \cdot \text{s}}} = \text{_____}$$

Solving for the entry length, we get:

$$L_e = 0.0575 \text{ Re } D_H = 0.0575 \left(\text{_____} \right) \left(\text{_____ m} \right)$$

$$L_e = 0.201 \text{ m}$$

Conclusion:

c) For turbulent flow, the entry length is relatively independent from Reynolds number and estimated to be:

$$L_e = 50 D_H$$

Substituting the hydraulic diameter of the channel into this equation yields:

$$L_e = 50 \left(\text{_____ m} \right)$$

$$L_e = \text{_____ m}$$

Conclusion:

Example 2.11-1: Compressible Flow of a Gas in a Pipe Line

A gas mixture of 12.5 mol % ethanol and 87.5 % water is leaving a boiler at a temperature of 400 °C and a pressure of 21.604 atm.

The ethanol/water mixture enters a reformer in a large-scale ethanol reforming plant at a rate of $425.96 \frac{\text{kg}}{\text{s}}$ through a commercial steel pipe with a length of 120 m and an inner diameter of 75 cm.

Determine the pressure of the ethanol/water vapor mixture entering the reformer if the viscosity of the gas is $1.705 \times 10^{-5} \text{ Pa} \cdot \text{s}$ [2].

Strategy

The pressure of the gas entering the reformer can be calculated using the equation for the pressure drop for isothermal compressible flow.

Solution

Equation 2.11-9 of Geankoplis can be solved for the pressure P_2 at the end of the pipe to yield:

$$P_2^2 = P_1^2 - \frac{f}{DM} - \frac{G^2}{2P_1^2} \ln \left(\frac{P_1}{P_2} \right)$$

where:

f = Fanning friction factor

ΔL = Length of the pipe, m

G = Mass flux of gas in the pipe, $\frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$

R = Gas constant, $\frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}}$

D = Inner diameter of the pipe

M = Molecular weight of the gas flowing through the pipe, $\frac{\text{kg}}{\text{mol}}$

P_1 = Pressure at the beginning of the pipe segment, Pa

P_2 = Pressure at the end of the pipe segment, Pa

2. DOE Hydrogen Program: DOE H2A Analysis Production Case Studies, http://www.hydrogen.energy.gov/h2a_prod_studies.html. Accessed: July 2010.

To use this equation, the friction factor and the mass flux of the ethanol mixture must be calculated. Before determining the friction factor we need to calculate the relative roughness of the pipe and the Reynolds number. Thus,

$$Re = \frac{Dv\rho}{\mu}$$

The velocity of the gas can be calculated with the following equation:

$$v = \frac{\dot{V}}{A}$$

However, to determine the volumetric flow rate, we need to divide the mass flow rate by the density of the fluid. The density of this mixture is calculated using ideal gas law equation of state.

$$\rho = \frac{\text{mass}}{\text{volume}}$$

Substituting the corresponding quantities into this equation, after calculating the molecular weight of the gas mixture, we get:

$$M = x_{\text{Ethanol}} M_{\text{Ethanol}} + x_{\text{H}_2\text{O}} M_{\text{H}_2\text{O}}$$

$$M = 0.125 \frac{\text{mol ethanol}}{\text{mol}} \left(\frac{\text{g}}{\text{mol ethanol}} \right) + 0.875 \frac{\text{mol H}_2\text{O}}{\text{mol}} \left(\frac{\text{g}}{\text{mol H}_2\text{O}} \right)$$

$$M = \frac{\text{g}}{\text{mol}}$$

$$\rho = \frac{(21.604 \text{ atm}) \left(21.5 \frac{\text{g}}{\text{mol}} \right) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right)}{\left(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \right) (673.15 \text{ K}) \left(\frac{1 \text{ m}^3}{1000 \text{ L}} \right)}$$

$$\rho = \frac{\text{kg}}{\text{m}^3}$$

Now the volumetric flow rate can be determined as follows:

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{425.96 \frac{\text{kg}}{\text{s}}}{\frac{\text{kg}}{\text{m}^3}} = \frac{\text{m}^3}{\text{s}}$$

Substituting this value into the equation for the velocity of the fluid yields:

$$v = \frac{\dot{V}}{A} = \frac{\frac{\text{m}^3}{\text{s}}}{\left[\frac{\pi \left(\frac{\text{m}}{4} \right)^2}{4} \right]} = \frac{\text{m}}{\text{s}}$$

The value of the Reynolds number required to determine the friction factor can now be calculated as shown below:

$$\text{Re} = \frac{(0.75 \text{ m}) \left(\frac{\text{m}}{\text{s}} \right) \left(\frac{\text{kg}}{\text{m}^3} \right)}{\left(\frac{\text{kg}}{\text{m} \cdot \text{s}} \right)}$$

$$\text{Re} = \underline{\hspace{2cm}}$$

The relative roughness $\frac{\epsilon}{D}$ of a commercial steel pipe is given by:

$$\frac{\epsilon}{D} = \frac{\text{m}}{\text{m}} = 6 \times 10^{-5}$$

The friction factor can be obtained from Figure 6-9 of *Perry's Chemical Engineers' Handbook*, 8th Edition to be:

$$f = \underline{\hspace{2cm}}$$

The only value left to be calculated before being able to solve for the pressure of the gas entering the reformer is the mass flux, defined as:

$$G = \frac{\dot{m}}{A}$$

Substituting the mass flow rate and the cross-sectional area of the pipe into this equation, gives:

$$G = \frac{425.96 \frac{\text{kg}}{\text{s}}}{\left[\frac{\pi \left(\frac{\text{m}}{4} \right)^2}{4} \right]} = \frac{425.96 \frac{\text{kg}}{\text{s}}}{\text{m}^2} = \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$

Finally we can substitute all the corresponding values into the equation for P_2^2 to get:

$$P_2^2 = \left[21.604 \text{ atm} \left(\frac{101325 \text{ Pa}}{1 \text{ atm}} \right) \right]^2 - \frac{4 \left(\frac{\text{kg}}{\text{m}^2 \cdot \text{s}} \right)^2 \left(8.314 \frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}} \right) (673.15)}{(0.75 \text{ m}) \left(\frac{\text{g}}{\text{mol}} \right) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right)}$$

$$- \frac{2 \left(\frac{\text{kg}}{\text{m}^2 \cdot \text{s}} \right)^2 \left(8.314 \frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}} \right) (673.15)}{\left(\frac{\text{g}}{\text{mol}} \right) \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right)} \ln \left[\frac{21.604 \text{ atm} \left(\frac{101325 \text{ Pa}}{1 \text{ atm}} \right)}{P_2} \right]$$

$$P_2^2 = \text{Pa}^2 - 4.178 \times 10^{11} \text{ Pa}^2 - \text{Pa}^2 \ln \left(\frac{\text{Pa}}{P_2} \right)$$

This equation can be solved using computer software or trial and error to yield:

$$P_2 \approx \text{Pa}$$

Example 2.11-2: Maximum Flow for Compressible Flow of a Gas

Determine the maximum velocity that can be obtained for the ethanol/water mixture from Example 2.11-1 and compare to the actual velocity of the fluid fed to the ethanol reformer.

Strategy

The maximum velocity of the fluid is obtained using the definition of the velocity of sound in an isothermal fluid.

Solution

The maximum velocity of the ethanol/water mixture can be determined using Equation 2.11-12 of Geankoplis, as shown below:

$$v_{\max} = \sqrt{\frac{RT}{M}}$$

Substituting the ideal gas constant, as well as the temperature of the gas in the pipe lines and the molecular weight into this equation yields:

$$v_{\max} = \sqrt{\frac{\left(\frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}}\right)(673.15 \text{ K})}{\left(\frac{\text{g}}{\text{mol}}\right)\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right)}}$$

$$v_{\max} = \frac{\text{m}}{\text{s}}$$

The velocity of this gas in the process at the entrance to the ethanol reformer is given by Equation 2.11-13 of Geankoplis:

$$v_2 = \frac{RTG}{P_2 M}$$

Substituting the pressure found in Example 2.11-1 and the mass flux of ethanol vapor, we find that the velocity is given by:

$$v_2 = \frac{\left(\frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}}\right)(673.15 \text{ K})\left(\frac{\text{kg}}{\text{m}^2 \cdot \text{s}}\right)}{\left(\frac{\text{Pa}}{\text{Pa}}\right)\left(\frac{\text{g}}{\text{mol}}\right)\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right)}}$$

$$v_2 = \frac{\text{m}}{\text{s}}$$