Chapter 8
Balances on Nonreactive Processes

Name: ____________
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Chapter 8 will cover procedures to determine changes in internal energy, ΔU or enthalpy, ΔH by performing energy balances when the values of $\hat{H}$ and $\hat{U}$ are not tabulated for all the species involved in a process. The following problems build upon the fundamentals covered in your text as applied to hydrogen processing, hydrogen as an energy carrier, and the use of hydrogen in fuel cells.

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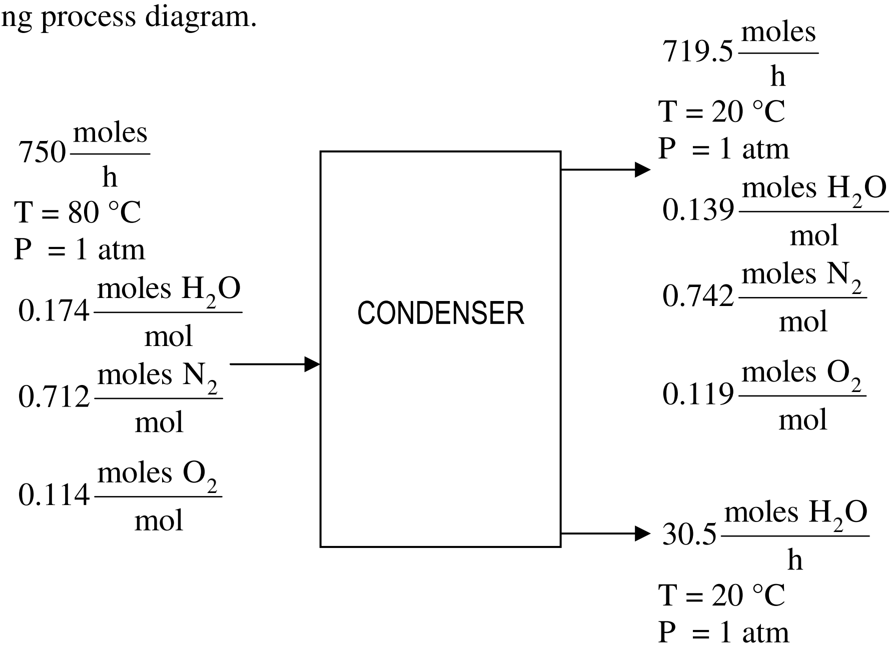
Example 8.1-1 Energy Balance on a Condenser

Water is being partially condensed out of the exhaust gases from a polymer-electrolyte membrane fuel cell. Calculate the amount of heat required to condense the water as described in the following process diagram.



Strategy

In order to solve this problem, we need to perform energy balances on the condenser. To determine the enthalpies required to complete the energy balance, we will select a reference state for each of the components in this process.

Solution

For this process, the change in the potential and kinetic energy will be neglected. In addition, since there are no moving parts in the condenser, the shaft work will be also neglected. Hence, the energy balance equation is given by:

$$\dot{Q} = \Delta\dot{H} = \sum_{\text{out}} \dot{n}_i \hat{H}_i - \sum_{\text{in}} \dot{n}_i \hat{H}_i$$

In order to facilitate the enthalpy calculations, the following reference states for each component in this process will be selected:

Liquid water @ T = 20°C, P = 1 atm
Nitrogen gas @ T = 20°C, P = 1 atm
Oxygen gas @ T = 20°C, P = 1 atm

The flow rates and enthalpies of the species in this process in both the inlet and outlet streams are shown in table below.

Species	$\dot{n}_{in}, \frac{mol}{h}$	$\hat{H}_{in}$	$\dot{n}_{out}, \frac{mol}{h}$	$\hat{H}_{out}$
H_2O (l)	-	-	________	0
H_2O (g)	________	$\hat{H}_1$	100	$\hat{H}_4$
N_2 (g)	534	$\hat{H}_2$	________	0
O_2 (g)	________	$\hat{H}_3$	85.5	0

There are some observations that can be made from this table:

> *Since there is no liquid water entering the condenser, the corresponding cells were marked out using dashes.
> *The flow rates were obtained by multiplying the molar fraction of each component by the total flow rate of the corresponding stream.
> *The enthalpy of the liquid water exiting the system was set to zero since it is leaving the system at the reference conditions. This also occurs for the nitrogen and oxygen leaving the system.
> *The unknown enthalpies were labeled and will be calculated to solve this problem.

To calculate the 4 unknown enthalpies in the table, hypothetical process paths from the reference states will be constructed.

When a liquid or solid undergoes a change in pressure, its enthalpy can be calculated as follows:

$$\Delta\hat{H} = \hat{V}\Delta P$$

where $\hat{V}$ is the specific volume of the liquid or solid.

Since there is no change in pressure between the actual conditions and the reference conditions for all the species in the condensation process, the changes in enthalpy as a result of a pressure difference will be equal to zero.

For a change in the temperature of a component, the enthalpy can be obtained by the following equation:

$$\Delta\hat{H} = \int_{T_i}^{T_f} C_p dT ,$$

where:

C_p = Specific heat of the component
T_f = Temperature of the component at its final conditions
T_i = Temperature of the component at its initial conditions

The expressions for the calculation of specific heat as a function of temperature can be obtained from Table B.2 of Felder & Rousseau. For each component, the equations are shown below:

$$C_{p,H_2O(l)}\left(\frac{kJ}{mol\cdot °C}\right)=7.54\times10^{-2}$$

$$C_{p,H_2O(g)}\left(\frac{kJ}{mol\cdot °C}\right)=\left(3.346\times10^{-2}\right)+\left(\_\_\_\_\_\_\_\_\_\_\_\_\right)T+\left(7.604\times10^{-9}\right)T^2-\left(3.593\times10^{-12}\right)T^3$$

$$C_{p,N_2(g)}\left(\frac{kJ}{mol\cdot °C}\right)=\left(\_\_\_\_\_\_\_\_\_\_\_\_\right)+\left(2.199\times10^{-6}\right)T+\left(5.723\times10^{-9}\right)T^2-\left(\_\_\_\_\_\_\_\_\_\_\_\_\right)T^3$$

$$C_{p,O_2(g)}\left(\frac{kJ}{mol\cdot °C}\right)=\left(2.91\times10^{-2}\right)+\left(\_\_\_\_\_\_\_\_\_\_\_\_\right)T-\left(\_\_\_\_\_\_\_\_\_\_\_\_\right)T^2+\left(1.311\times10^{-12}\right)T^3$$

The integration of these expressions with respect to temperature yields:

$$\int_{T_i}^{T_f} C_{p,H_2O(l)}dT=\left(7.54\times10^{-2}\right)T\Big|_{T_i}^{T_f}$$

$$\int_{T_i}^{T_f} C_{p,H_2O(g)}dT=\left(3.346\times10^{-2}\right)T+\left(3.44\times10^{-6}\right)T^2+\left(\_\_\_\_\_\_\_\_\_\_\_\_\right)T^3-\left(8.983\times10^{-13}\right)T^4\Big|_{T_i}^{T_f}$$

$$\int_{T_i}^{T_f} C_{p,N_2(g)}dT=\left(2.9\times10^{-2}\right)T+\left(\_\_\_\_\_\_\_\_\_\_\_\_\right)T^2+\left(1.908\times10^{-9}\right)T^3-\left(\_\_\_\_\_\_\_\_\_\_\_\_\right)T^4\Big|_{T_i}^{T_f}$$

$$\int_{T_i}^{T_f} C_{p,O_2(g)}dT=\left(\_\_\_\_\_\_\_\_\_\_\_\_\right)T+\left(\_\_\_\_\_\_\_\_\_\_\_\_\right)T^2-\left(2.025\times10^{-9}\right)T^3+\left(3.278\times10^{-13}\right)T^4\Big|_{T_i}^{T_f}$$

The units for the specific heat are $\frac{kJ}{mol\cdot K}$ or $\frac{kJ}{mol\cdot °C}$

If there is a phase change for any of the species present in the process, the change of enthalpy is equal to the heat of vaporization $\Delta\hat{H}_v$.

The only component in this condensation process which is experiencing a phase change is water. Thus, the enthalpy of vaporization for water at the boiling point can be obtained from Table B.1 of Felder & Rousseau:

$$\Delta\hat{H}_{v,H_2O} = \underline{\hspace{2.5cm}}\ \frac{kJ}{mol}$$

The enthalpy of the water vapor entering the process will be calculated by the addition of the following changes in enthalpy:

-Change of enthalpy of the liquid water from T_{ref} to the boiling point temperature T_{bp}.
-Heat of vaporization from liquid to vapor.
-Change of enthalpy of the vapor water from the boiling temperature T_{bp} to the inlet temperature T_{in}

Writing these enthalpies as equations, we have

$$\hat{H}_1 = \int_{T_{ref}}^{T_{bp}} C_{p,H_2O(l)}dT + \Delta H_v + \int_{Tbp}^{T_{in}} C_{p,H_2O(l)}dT$$

substituting the corresponding expression and numerical values into this equation yields:

$$\begin{aligned}\hat{H}_1\left(\frac{kJ}{mol}\right) = & \left[\left(\ \underline{\hspace{2.5cm}}\ \right)(100°C-20°C)\right] + 40.656 + \left[\left(3.346\times10^{-2}\right)(80°C-100°C)\right] \\ & + \left\{\left(3.44\times10^{-6}\right)\left[\left(\ \underline{\hspace{1cm}}°C\right)^2 - \left(\ \underline{\hspace{1.2cm}}°C\right)^2\right]\right\} \\ & + \left\{\left(\ \underline{\hspace{3.2cm}}\ \right)\left[\left(\ \underline{\hspace{1cm}}°C\right)^3 - \left(\ \underline{\hspace{1.2cm}}°C\right)^3\right]\right\} \\ & - \left\{\left(8.983\times10^{-13}\right)\left[\left(\ \underline{\hspace{1cm}}°C\right)^4 - \left(\ \underline{\hspace{1cm}}°C\right)^4\right]\right\}\end{aligned}$$

$$\hat{H}_1 = 46.01\frac{kJ}{mol}$$

For the enthalpy of the water vapor exiting the condenser, a similar procedure is followed, only that the temperature at the final condition will be the temperature of the water vapor exiting the process:

$$\hat{H}_4 = \int_{T_{ref}}^{T_{bp}} C_{p,H_2O(l)}dT + \Delta H_v + \int_{Tbp}^{T_{out}} C_{p,H_2O(l)}dT$$

By entering the corresponding specific heat expressions and temperatures into this equation, we get:

$$\begin{aligned}\hat{H}_4\left(\frac{\text{kJ}}{\text{mol}}\right)=&\left[\left(7.54\times10^{-2}\right)(100°\text{C}-20°\text{C})\right]+40.656+\left[\left(3.346\times10^{-2}\right)(20°\text{C}-100°\text{C})\right]\\&+\left\{\left(3.44\times10^{-6}\right)\left[(\text{\_\_\_\_}°\text{C})^2-(\text{\_\_\_\_\_\_}°\text{C})^2\right]\right\}\\&+\left\{(\text{\_\_\_\_\_\_\_\_\_\_})\left[(\text{\_\_\_\_}°\text{C})^3-(\text{\_\_\_\_\_}°\text{C})^3\right]\right\}\\&-\left\{\left(8.983\times10^{-13}\right)\left[(\text{\_\_\_\_}°\text{C})^4-(\text{\_\_\_\_\_\_}°\text{C})^4\right]\right\}\end{aligned}$$

$$\hat{H}_4=43.98\frac{\text{kJ}}{\text{mol}}$$

The enthalpy of the nitrogen and oxygen in the process will be obtained by calculating the change in enthalpy from the reference temperature to the temperature at the final conditions. For nitrogen and oxygen, the heat of vaporization will not be considered because there is no phase change for these gases. This can be written as follows

$$\hat{H}_2=\int_{T_{ref}}^{T_{in}}C_{p,N_2(g)}dT$$

$$\hat{H}_3=\int_{T_{ref}}^{T_{in}}C_{p,O_2(g)}dT$$

Substituting the integrated expressions for the specific heat of nitrogen and oxygen into the corresponding equation yields:

$$\begin{aligned}\hat{H}_2\left(\frac{\text{kJ}}{\text{mol}}\right)=&\left[\left(2.9\times10^{-2}\right)(80°\text{C}-20°\text{C})\right]+\left\{\left(1.1\times10^{-6}\right)\left[(80°\text{C})^2-(20°\text{C})^2\right]\right\}\\&+\left\{(\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_})\left[(80°\text{C})^3-(20°\text{C})^3\right]\right\}\\&-\left\{(\text{\_\_\_\_\_\_\_\_\_\_\_\_})\left[(80°\text{C})^4-(20°\text{C})^4\right]\right\}\end{aligned}$$

$$\hat{H}_2=\text{\_\_\_\_\_\_}\frac{\text{kJ}}{\text{mol}}$$

$$\hat{H}_3\left(\frac{\text{kJ}}{\text{mol}}\right)=\left[(\text{\_\_\_\_\_\_\_\_\_\_})(\text{\_\_\_\_}°\text{C}-\text{\_\_\_\_}°\text{C})\right]$$

$$+\left\{(\text{\_\_\_\_\_\_\_\_\_\_})\left[(\text{\_\_\_\_}°\text{C})^2-(\text{\_\_\_\_}°\text{C})^2\right]\right\}$$

$$-\left\{\left(2.025\times10^{-9}\right)\left[(\text{\_\_\_\_}°\text{C})^3-(\text{\_\_\_\_}°\text{C})^3\right]\right\}$$

$$+\left\{\left(3.278\times10^{-13}\right)\left[(\text{\_\_\_\_}°\text{C})^4-(\text{\_\_\_\_}°\text{C})^4\right]\right\}$$

$$\hat{H}_3=\text{\_\_\_\_\_\_\_}\frac{\text{kJ}}{\text{mol}}$$

The general balance equation can be rewritten as follows, according to the number of components present in the condensation process (neglecting zero enthalpies):

$$\dot{Q}=\Delta\dot{H}=\dot{n}_4\hat{H}_4+\cancel{\dot{n}_5\hat{H}_5}+\cancel{\dot{n}_6\hat{H}_6}-\dot{n}_1\hat{H}_1-\dot{n}_2\hat{H}_2-\dot{n}_3\hat{H}_3$$

Substituting the known flow rates and enthalpies into this equation yields:

$$\dot{Q}=\Delta\dot{H}=\left(\text{\_\_\_\_\_}\frac{\text{mol H}_2\text{O}}{\text{h}}\right)\left(43.98\frac{\text{kJ}}{\text{mol H}_2\text{O}}\right)-\left(\text{\_\_\_\_\_}\frac{\text{mol H}_2\text{O}}{\text{h}}\right)\left(46.01\frac{\text{kJ}}{\text{mol H}_2\text{O}}\right)$$

$$-\left(534\frac{\text{mol N}_2}{\text{h}}\right)\left(\text{\_\_\_\_\_}\frac{\text{kJ}}{\text{mol N}_2}\right)-\left(85.5\frac{\text{mol O}_2}{\text{h}}\right)\left(\text{\_\_\_\_\_}\frac{\text{kJ}}{\text{mol O}_2}\right)$$

$$\dot{Q}=-\text{\_\_\_\_\_\_\_\_\_}\ \frac{\text{kJ}}{\text{h}}\left(\frac{1\text{ h}}{3600\text{ s}}\right)$$

$$\boxed{\dot{Q}=\text{\_\_\_\_\_\_\_\_\_\_}\ \text{kW}}$$

The negative sign on the result indicates that this amount of heat must be removed from the process in order to achieve the condensation of the amount of water shown in the process diagram.

Example 8.3-1 Evaluation of an Internal Energy Change from Tabulated Heat Capacity

Hydrogen produced by steam-methane reforming exits at a pressure of 1.6 MPa and a temperature of 855°C. The hydrogen is stored in a metal hydride tank with a volume of 70 m^3 and must be cooled down to a temperature of 30°C before being fed to a proton exchange membrane fuel cell. Determine the amount of heat required to cool down the hydrogen obtained from the steam-methane reforming process.

Assume that the constant-volume heat capacity of hydrogen is given by the following equation (Example 8.3-2 shows how to calculate this):

$$C_v\left(\frac{\text{kJ}}{\text{mol}\cdot{}^\circ\text{C}}\right) = (2.053\times10^{-2}) + (7.650\times10^{-8})T + (3.288\times10^{-9})T^2 - (8.698\times10^{-13})T^3$$

Strategy

Since the gas is being cooled inside a constant-volume tank, the heat transferred is equal to the change in internal energy of the gas.

Solution

From the first law of thermodynamics applied for a closed system, the heat transferred is given by:

$$Q = \Delta U$$

The change in internal energy is defined by Equation 8.3-6 of Felder & Rousseau:

$$\Delta\hat{U} = \int_{T_1}^{T_2} C_v(T)\, dT$$

Substituting the heat capacity equation into the integral:

$$\Delta\hat{U} = \int_{T_1}^{T_2}\left[(2.053\times10^{-2}) + (7.650\times10^{-8})T + (3.288\times10^{-9})T^2 - (8.698\times10^{-13})T^3\right] dT$$

$$\Delta\hat{U} = \left[(2.053\times10^{-2})T + (\text{\_\_\_\_\_\_\_\_\_\_\_\_})T^2 + (1.096\times10^{-9})T^3 - (\text{\_\_\_\_\_\_\_\_\_\_\_\_})T^4\right]_{T_1}^{T_2}$$

Evaluating the integral at the initial and final temperatures gives:

$$\Delta\hat{U} = (2.053\times10^{-2})[\_\_\_\_ ^\circ\text{C} - \_\_\_\_\_\_ ^\circ\text{C}] + (\ \_\_\_\_\_\_\_\_\_\_\ )\left[(\ \_\_\_\_ ^\circ\text{C})^2 - (\ \_\_\_\_\_\_ ^\circ\text{C})^2\right]$$
$$+ (1.096\times10^{-9})\left[(\ \_\_\_\_ ^\circ\text{C})^3 - (\ \_\_\_\_\_\_ ^\circ\text{C})^3\right]$$
$$-(\ \_\_\_\_\_\_\_\_\_\_\ )\left[(\ \_\_\_\_ ^\circ\text{C})^4 - (\ \_\_\_\_\_\_ ^\circ\text{C})^4\right]$$

$$\Delta\hat{U} = \_\_\_\_\_\_\_\_ \frac{\text{kJ}}{\text{mol}}$$

To determine the amount of energy, the molar internal energy must be multiplied by the number of moles of hydrogen, which can be calculated using the ideal gas equation:

$$PV = nRT$$

Solving for the number of moles n and entering the known quantities in this equation yields:

$$n = \frac{PV}{RT} = \frac{(\ \_\_\_\_\_\_\_\_ \text{ Pa})(70 \text{ m}^3)}{\left(8.314 \frac{\text{Pa}\cdot\text{m}^3}{\text{mol}\cdot\text{K}}\right)(\ \_\_\_\_\_\_ \text{ K})}$$

$$n = \_\_\_\_\_\_\_\_ \text{ moles}$$

Thus, the total heat transferred can be calculated as follows:

$$Q = \Delta U = n\Delta\hat{U}$$

Substituting the molar internal energy and the number of moles of hydrogen in the storage tank into this equation we get:

$$Q = \_\_\_\_\_\_\_\_ \text{ moles}\left(\_\_\_\_\_\_\_\_ \frac{\text{kJ}}{\text{mol}}\right)$$

$$\boxed{Q = \_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ kJ}}$$

Example 8.3-2 Heating and Cooling of an Ideal Gas

a) A large-scale coal gasification plant is producing synthesis gas at a temperature of 650 K and a pressure of 6.3 MPa. The rate of production from this process is $1.2\times10^6 \frac{\text{kg H}_2}{\text{day}}$. The hydrogen is produced for use in a Phosphoric-Acid Fuel cell, which operates at a temperature of 220 °C. Determine the heat transferred from the cooling of hydrogen.

Strategy

The energy balance equation must be applied in order to solve this problem. Since the process described in the problem statement represents an open system, the appropriate form of the first law of thermodynamics can be used to determine the heat transfer rate.

Solution

Equation 8.3-2 of Felder & Rousseau gives the heat transfer rate for an open system (neglecting shaft work, kinetic and potential energies):

$$\dot{Q} = \Delta\dot{H}$$

The definition of the molar change in enthalpy, $\Delta\hat{H}$ is given by the following equation:

$$\Delta\hat{H} = \int_{T_1}^{T_2} C_p(T)dT$$

The expression for the C_p of hydrogen as a function of temperature can be obtained from Table B.2 of Felder & Rousseau, and it is given by:

$$C_p\left(\frac{\text{kJ}}{\text{mol}\cdot{}^\circ\text{C}}\right) = (2.884\times10^{-2}) + (\underline{\hspace{3cm}})T + (\underline{\hspace{3cm}})T^2 - (8.698\times10^{-13})T^3$$

Substituting the heat capacity equation at constant pressure in the definition of the change in molar enthalpy gives:

$$\Delta\hat{H} = \int_{T_1}^{T_2}\left[(2.884\times10^{-2}) + (\underline{\hspace{3cm}})T + (\underline{\hspace{3cm}})T^2 - (8.698\times10^{-13})T^3\right]dT$$

Integrating this expression, we have:

$$\Delta\hat{H} = \left[(2.884\times10^{-2})T + (\underline{\hspace{3cm}})T^2 + (1.096\times10^{-9})T^3 - (\underline{\hspace{3cm}})T^4\right]\Bigg|_{T_1}^{T_2}$$

Evaluating the integral at its limits yields:

$$\Delta\hat{H} = (2.884\times10^{-2})\left[220°\text{C} - 377°\text{C}\right] + (\underline{\qquad\qquad})\left[(220°\text{C})^2 - (377°\text{C})^2\right]$$
$$+ (1.096\times10^{-9})\left[(220°\text{C})^3 - (377°\text{C})^3\right] - (\underline{\qquad\qquad})\left[(220°\text{C})^4 - (377°\text{C})^4\right]$$

$$\Delta\hat{H} = \underline{\qquad\quad}\ \frac{\text{kJ}}{\text{mol}}$$

The heat transfer rate can now be obtained by multiplying the change in molar enthalpy by the molar flow rate of hydrogen. Thus,

$$\dot{Q} = \Delta\dot{H} = \dot{n}\Delta\hat{H}$$

Entering the numeric values into this equation and applying conversion factors, we get:

$$\dot{Q} = \left(\underline{\qquad\qquad}\frac{\text{kg}}{\text{day}}\right)\left(\underline{\qquad\quad}\frac{\text{kJ}}{\text{mol}}\right)\cdot\left(\frac{1\ \text{kmol}\ H_2}{\underline{\quad\ }\ \text{kg}\ H_2}\right)\left(\frac{1000\ \text{moles}}{1\ \text{kmol}}\right)\left(\frac{1\ \text{day}}{\underline{\qquad\qquad}\ \text{s}}\right)$$

$$\boxed{\dot{Q} = -3.2\times10^4\ \underline{\qquad}}$$

b) In a steam-methane reforming process, 3000 kg of steam at 100 °C are to be stored in a tank and heated to a temperature of 210 °C, before entering the reaction chamber to produce hydrogen. Determine the amount of heat that must be applied to the steam to reach 210 °C.

Strategy

To solve this problem we need to use the energy balance equation for a closed system. To calculate the required amount of heat, the heat capacity of steam at constant volume must be obtained from the heat capacity at constant pressure.

Solution

Since the steam is being stored in a tank, the process described in the problem statement is occurring at constant volume. Hence, the amount of heat transferred is given by:

$$Q = \Delta U$$

The change in the molar internal energy of the steam can be obtained by integrating the heat capacity equation at constant volume with respect to temperature:

$$\Delta\hat{U} = \int_{T_1}^{T_2} C_v(T)\,dT$$

Table B.2 of Felder & Rousseau only gives the equation for the heat capacity at constant pressure, C_p. However, if we assume that the steam in this process shows ideal gas behavior, the following equation can be used to determine C_v:

$$C_v = C_p - R$$

Substituting the expression for C_p as a function of temperature into this equation, and the universal gas constant into this equation, we get:

$$C_v = \left[(\underline{\qquad\qquad}) + (6.880 \times 10^{-6})T + (\underline{\qquad\qquad})T^2 - (\underline{\qquad\qquad})T^3 \right] \frac{\text{kJ}}{\text{mol} \cdot {}^\circ\text{C}}$$
$$- \left(8.314 \times 10^{-3} \frac{\text{kJ}}{\text{mol} \cdot {}^\circ\text{C}} \right)$$

The expression for the heat capacity at constant volume can now be obtained by subtracting the universal gas constant from the C_p equation to obtain:

$$C_v \left(\frac{\text{kJ}}{\text{mol} \cdot {}^\circ\text{C}} \right) = (\underline{\qquad\qquad}) + (\underline{\qquad\qquad})T + (7.604 \times 10^{-9})T^2$$
$$- (\underline{\qquad\qquad})T^3 \frac{\text{kJ}}{\text{mol} \cdot {}^\circ\text{C}}$$

Note that the universal gas constant can have either units of $\frac{\text{kJ}}{\text{mol} \cdot {}^\circ\text{C}}$ or $\frac{\text{kJ}}{\text{mol} \cdot \text{K}}$ because the absolute value of the temperature difference is the same for both degrees Kelvin and degrees Celsius.

Now that we have the C_v equation as a function of temperature, we can substitute it into the definition of the change in molar internal energy and integrate it to get:

$$\Delta\hat{U} \left(\frac{\text{kJ}}{\text{mol}} \right) = \left[(\underline{\qquad\qquad})T + (\underline{\qquad\qquad})T^2 + (2.535 \times 10^{-9})T^3 - (\underline{\qquad\qquad})T^4 \right]_{T_1}^{T_2}$$

Applying the limits of integration and substituting the temperature values yields:

$$\Delta\hat{U}\left(\frac{\text{kJ}}{\text{mol}}\right)=(\text{\_\_\_\_\_\_\_\_})[\text{\_\_\_\_}{}^{\circ}\text{C}-\text{\_\_\_\_}{}^{\circ}\text{C}]+(\text{\_\_\_\_\_\_\_\_})\left[(\text{\_\_\_\_}{}^{\circ}\text{C})^{2}-(\text{\_\_\_\_}{}^{\circ}\text{C})^{2}\right]$$
$$+(2.535\times10^{-9})\left[(\text{\_\_\_\_}{}^{\circ}\text{C})^{3}-(\text{\_\_\_\_}{}^{\circ}\text{C})^{3}\right]-(\text{\_\_\_\_\_\_\_\_})\left[(\text{\_\_\_\_}{}^{\circ}\text{C})^{4}-(\text{\_\_\_\_}{}^{\circ}\text{C})^{4}\right]$$

$$\Delta\hat{U}=2.903\frac{\text{kJ}}{\text{mol}}$$

To find the total amount of heat being transferred to the steam, the molar internal energy must be multiplied by the number of moles of steam in the tank. To calculate the number of moles, we divide the mass of steam by the molecular weight of water:

$$Q=\Delta U=\text{\_\_\_\_\_\_\_}\ \text{kg}\left(\frac{1\ \text{kmol}}{\text{\_\_\_\_}\ \text{kg}}\right)\left(\frac{1000\ \text{moles}}{1\ \text{kmol}}\right)\left(2.903\frac{\text{kJ}}{\text{mol}}\right)$$

$$\boxed{Q=\text{\_\_\_\_\_\_\_\_\_\_}\ \text{kJ}}$$

Example 8.3-4 Heat Capacity of a Mixture

A molten carbonate fuel cell is fed with a gas mixture of carbon dioxide and oxygen at a rate of 31 $\frac{\text{mol}}{\text{hr}}$ and a temperature of 650 °C. Determine the heat capacity of this mixture if the molar ratio of oxygen to carbon dioxide is 0.5.

Strategy

The solution to this problem can be obtained by calculating the heat capacity of each gas as a function of temperature.

Solution

To determine the heat capacity of both carbon dioxide and oxygen, the equations must be built from the parameters shown in Table B.2 of Felder & Rousseau:

$$C_{p,O_2(g)}\left(\frac{\text{kJ}}{\text{mol}\cdot{}^\circ\text{C}}\right)=(\ \underline{\qquad\qquad}\ )+\left(1.158\times10^{-5}\right)T-(\ \underline{\qquad\qquad}\ )T^2+\left(1.311\times10^{-12}\right)T^3$$

$$C_{p,CO_2(g)}\left(\frac{\text{kJ}}{\text{mol}\cdot{}^\circ\text{C}}\right)=\left(3.611\times10^{-2}\right)+(\ \underline{\qquad\qquad}\ )T-(\ \underline{\qquad\qquad}\ )T^2+(\ \underline{\qquad\qquad}\ )T^3$$

When the heat capacity of a mixture is to be determined, the following equation should be used:

$$C_{p,mix}(T)=\sum_{\substack{\text{all mixture}\\\text{components}}}\left[y_i, C_{p,i}(T)\right]$$

The molar fraction of each gas can be calculated using the ratio of moles of oxygen to moles of carbon dioxide.

$$y_{O_2}=0.5y_{CO_2}$$

In addition, it is known that the constituents of the gas being fed into the fuel cell are carbon dioxide and oxygen. Thus,

$$y_{O_2}+y_{CO_2}=1$$

Substituting the equation previously solved for the molar fraction of oxygen into this equation and solving for y_{CO2}, we get:

$$0.5y_{CO_2} + y_{CO_2} = 1$$

$$y_{CO_2} = \frac{1}{1.5}$$

$$y_{CO_2} = \underline{\qquad\qquad} \frac{\text{moles } CO_2}{\text{mol}}$$

This value can be entered into the equation for the molar fraction of oxygen y_{O2} in terms of the molar fraction of carbon dioxide y_{CO2} to get:

$$y_{O_2} = \left(0.5 \frac{\text{moles } O_2}{\text{moles } CO_2}\right)\left(\underline{\qquad\quad} \frac{\text{moles } CO_2}{\text{mol}}\right)$$

$$y_{O_2} = \underline{\qquad\qquad} \frac{\text{moles } O_2}{\text{mol}}$$

Now, both of the heat capacity equations as a function of temperature must be multiplied by the corresponding mol fraction:

$$C_{p,mix}(T) = y_{O_2}, C_{p,O_2}(T) + y_{CO_2}, C_{p,CO_2}(T)$$

Substituting the heat capacity expressions into this equation yields:

$$\begin{aligned} C_{p,mix}(T) = {} & \underline{\qquad\quad} \frac{\text{moles } O_2}{\text{mol}} \left[\left(\underline{\qquad\qquad}\right) + \left(1.158 \times 10^{-5}\right)T - \left(\underline{\qquad\qquad}\right)T^2 + \left(1.311 \times 10^{-12}\right)T^3\right] \\ & + \underline{\qquad\quad} \frac{\text{moles } CO_2}{\text{mol}} \left[\left(3.611 \times 10^{-2}\right) + \left(\underline{\qquad\qquad}\right)T - \left(\underline{\qquad\qquad}\right)T^2 + \left(\underline{\qquad\qquad}\right)T^3\right] \end{aligned}$$

$$\begin{aligned} C_{p,mix}(T) = {} & \left[\left(\underline{\qquad\qquad}\right) + \left(3.860 \times 10^{-6}\right)T - \left(\underline{\qquad\qquad}\right)T^2 + \left(4.370 \times 10^{-13}\right)T^3\right] \\ & + \left[\left(2.407 \times 10^{-2}\right) + \left(\underline{\qquad\qquad}\right)T - \left(\underline{\qquad\qquad}\right)T^2 + \left(\underline{\qquad\qquad}\right)T^3\right] \end{aligned}$$

This equation can be reduced by adding similar terms to get:

$$C_{p,mix}(T) = \left(3.377 \times 10^{-3}\right) + \left(3.208 \times 10^{-5}\right)T - \left(\underline{\qquad\qquad}\right)T^2 + \left(\underline{\qquad\qquad}\right)T^3$$

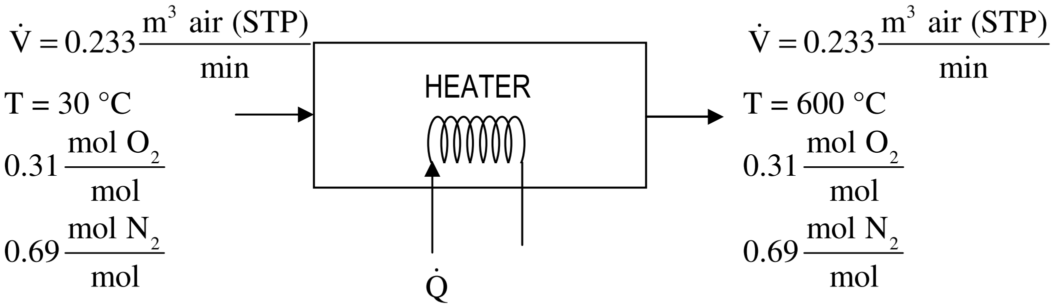
The heat capacity of the mixture can now be calculated by entering the operating temperature of the fuel cell, which is 650 °C:

$$C_{p,mix}(T) = \left(3.377\times10^{-3}\right) + \left(3.208\times10^{-5}\right)(650\ °C) - (\ \underline{\qquad\qquad}\ )(650\ °C)^2 + (\ \underline{\qquad\qquad}\ )(650\ °C)^3$$

$$\boxed{C_{p,mix} = 1.672\times10^{-2}\ \frac{kJ}{mol\cdot K}}$$

8.3-5 Energy Balance on a Gas Preheater

The cathode side of a solid oxide fuel cell is fed with air which is heated from 30 °C to 600 °C before entering the fuel cell. The air is entering at a rate of $0.233 \frac{\text{m}^3\text{(STP)}}{\text{min}}$ and contains 31 mole % of oxygen and the rest of nitrogen. Determine the amount of heat transferred to the air. A diagram of the heating process is shown below:



Strategy

The strategy for solving this problem is to use energy balances and determine the enthalpies of the components of air in this process by selecting reference states. In addition, the flow rate given in the problem statement will have to be converted to the operating conditions of the process.

Solution

The following table summarizes the species involved in the air heating process for the fuel cell.

Substance	$\dot{n}_{in}, \frac{\text{mol}}{\text{min}}$	$\hat{H}_{in}, \frac{\text{kJ}}{\text{mol}}$	$\dot{n}_{out}, \frac{\text{mol}}{\text{min}}$	$\hat{H}_{out}, \frac{\text{kJ}}{\text{mol}}$
O_2	$\dot{n}_1$	$\hat{H}_1$	$\dot{n}_3$	$\hat{H}_3$
N_2	$\dot{n}_2$	$\hat{H}_2$	$\dot{n}_4$	$\hat{H}_4$

The molar flow rate can be obtained by the following equation:

$$\dot{n} = \frac{\dot{V}_{std}}{\hat{V}_{std}},$$

where:

$\dot{V}_{std}$ =Volumetric flow rate at standard temperature and pressure

$\hat{V}_{std}$ =Molar volume of the gas at standard temperature and pressure

Substituting the known values into the equation for the molar flow rate, we get:

$$\dot{n} = \frac{0.233\,\frac{m^3}{min}}{\_\_\_\_\_\_\_\_\frac{m^3}{mol}} = \_\_\_\_\_\frac{mol}{min}$$

Now the flow rate of each one of the gases being heated can be calculated as follows:

$$\dot{n}_i = y_i\dot{n}$$

Entering numeric values into this equation will yield:

$$\dot{n}_{O_2} = \left(\_\_\_\_\frac{mol\ O_2}{mol}\right)\left(\_\_\_\_\_\frac{mol}{min}\right) = 3.22\frac{mol\ O_2}{min}$$

$$\dot{n}_{N_2} = \left(\_\_\_\_\frac{mol\ N_2}{mol}\right)\left(\_\_\_\_\_\frac{mol}{min}\right) = \_\_\_\_\_\frac{mol\ N_2}{min}$$

The energy balance equation omitting the kinetic and potential energy changes and the shaft work terms will be given by:

$$\dot{Q} = \Delta\dot{H} = \sum_{out}\dot{n}_i\hat{H}_i - \sum_{in}\dot{n}_i\hat{H}_i$$

To determine the unknown enthalpies in this problem, the selected reference state for air will be:

Air @ P = 1 atm, T = 30 °C

Since the air is entering the heater at the same conditions as the reference of state of air, the enthalpies H_1 and H_2 will be equal to zero.

The updated table will look as follows:

Substance	$\dot{n}_{in}, \frac{mol}{min}$	$\hat{H}_{in}, \frac{kJ}{mol}$	$\dot{n}_{out}, \frac{mol}{min}$	$\hat{H}_{out}, \frac{kJ}{mol}$
O_2	3.22	0	_____	$\hat{H}_3$
N_2	_____	0	_____	$\hat{H}_4$

The remaining unknown enthalpies will be calculated based on the selected reference temperature, using the heat capacity of each gas as a function of temperature:

$$\hat{H}_3 = \int_{T_{ref}}^{T_{out}} C_{p,O_2}$$
$$\hat{H}_3 = \int_{T_{ref}}^{T_{out}} (\underline{\qquad\qquad}) + (\underline{\qquad\qquad})T - (\underline{\qquad\qquad})T^2 + \left(1.311\times10^{-12}\right)T^3$$
$$\hat{H}_3 = (\underline{\qquad\qquad})T + (\underline{\qquad\qquad})T^2 - (\underline{\qquad\qquad})T^3 + \left(3.278\times10^{-13}\right)T^4 \Big|_{T_{ref}}^{T_{out}}$$

The same procedure is followed for the enthalpy of nitrogen in the air exiting the heater:

$$\hat{H}_4 = \int_{T_{ref}}^{T_{out}} C_{p,N_2}$$
$$\hat{H}_4 = \int_{T_{ref}}^{T_{out}} (\underline{\qquad\qquad}) + (\underline{\qquad\qquad})T + (\underline{\qquad\qquad})T^2 - (\underline{\qquad\qquad})T^3$$
$$\hat{H}_4 = (\underline{\qquad\qquad})T + (\underline{\qquad\qquad})T^2 + (\underline{\qquad\qquad})T^3 - (\underline{\qquad\qquad})T^4 \Big|_{T_{ref}}^{T_{out}}$$

Substituting the temperature values into these equations gives:

$$\hat{H}_3 = \left[(\underline{\qquad\qquad})(600°\mathrm{C} - 30°\mathrm{C})\right]\frac{\mathrm{kJ}}{\mathrm{mol}} + \left\{(\underline{\qquad\qquad})\left[(600°\mathrm{C})^2 - (30°\mathrm{C})^2\right]\right\}\frac{\mathrm{kJ}}{\mathrm{mol}}$$
$$-\left\{(\underline{\qquad\qquad})\left[(600°\mathrm{C})^3 - (30°\mathrm{C})^3\right]\right\}\frac{\mathrm{kJ}}{\mathrm{mol}} + \left\{\left(3.278\times10^{-13}\right)\left[(600°\mathrm{C})^4 - (30°\mathrm{C})^4\right]\right\}\frac{\mathrm{kJ}}{\mathrm{mol}}$$
$$\hat{H}_3 = \underline{\qquad\qquad}\frac{\mathrm{kJ}}{\mathrm{mol}}$$

$$\hat{H}_4 = \left[(\underline{\qquad\qquad})(600°\mathrm{C} - 30°\mathrm{C})\right]\frac{\mathrm{kJ}}{\mathrm{mol}} + \left\{(\underline{\qquad\qquad})\left[(600°\mathrm{C})^2 - (30°\mathrm{C})^2\right]\right\}\frac{\mathrm{kJ}}{\mathrm{mol}}$$
$$+\left\{(\underline{\qquad\qquad})\left[(600°\mathrm{C})^3 - (30°\mathrm{C})^3\right]\right\}\frac{\mathrm{kJ}}{\mathrm{mol}} - \left\{(\underline{\qquad\qquad})\left[(600°\mathrm{C})^4 - (30°\mathrm{C})^4\right]\right\}\frac{\mathrm{kJ}}{\mathrm{mol}}$$
$$\hat{H}_4 = \underline{\qquad\qquad}\frac{\mathrm{kJ}}{\mathrm{mol}}$$

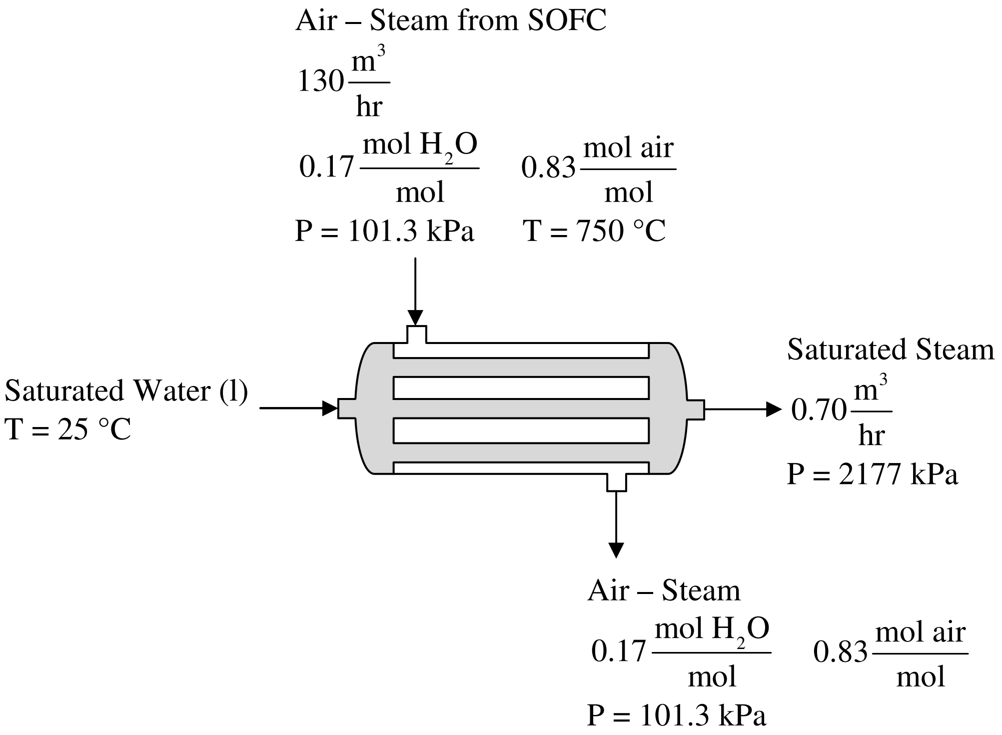
These enthalpy values and the molar flow rates of each gas can now be substituted into the energy balance equation to determine the amount of heat transferred to air:

$$\dot{Q} = \Delta\dot{H} = \dot{n}_3\hat{H}_3 + \dot{n}_4\hat{H}_4$$

$$\dot{Q} = \left[\left(3.22\frac{\mathrm{mol}}{\mathrm{min}}\right)\left(\underline{\qquad\qquad}\frac{\mathrm{kJ}}{\mathrm{mol}}\right) + \left(\underline{\qquad}\frac{\mathrm{mol}}{\mathrm{min}}\right)\left(\underline{\qquad\qquad}\frac{\mathrm{kJ}}{\mathrm{mol}}\right)\right]\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right)$$
$$\boxed{\dot{Q} = 3\ \mathrm{kW}}$$

8.3-6 Energy Balance on a Waste Heat Boiler

The exhaust gases from a solid-oxide fuel cell are leaving at a temperature of 750 °C and at standard atmospheric pressure. These gases are a mixture of steam and air and will be used to boil water according to the following diagram.



a) Determine the temperature of the outlet air/steam mixture stream. The selected reference states for the streams in this process are:

Air @ T = 750 °C, P = 101.3 kPa
Liquid water @ Triple point (0°C)

Strategy

The energy balances for an adiabatic system will be the key for solving this problem. The heat capacity equations as a function of temperature will be used to calculate the required temperature.

Solution

The energy balance equation for adiabatic operation, negligible potential and kinetic energies and no mechanical parts moving ($W_s = 0$) is reduced to:

$$\Delta \dot{H} = \sum_{out} \dot{n}_i \hat{H}_i - \sum_{in} \dot{n}_i \hat{H}_i = 0$$

The following table is showing the species involved in the boiling process and is showing the unknown flow rates and enthalpies.

Substance	$\dot{n}_{in}, \frac{mol}{hr}$	$\hat{H}_{in}, \frac{kJ}{mol}$	$\dot{n}_{out}, \frac{mol}{hr}$	$\hat{H}_{out}, \frac{kJ}{mol}$
O_2	$\dot{n}_1$	$\hat{H}_1$	$\dot{n}_5$	$\hat{H}_5$
N_2	$\dot{n}_2$	$\hat{H}_2$	$\dot{n}_6$	$\hat{H}_6$
H_2O (superheated steam from SOFC)	$\dot{n}_3$	$\hat{H}_3$	$\dot{n}_7$	$\hat{H}_7$
H_2O (saturated)	$\dot{n}_4$	$\hat{H}_4$	$\dot{n}_8$	$\hat{H}_8$

The molar flow rates of each species have to be determined since the process diagram is giving only the volumetric flow rates.

For the gas streams, the molar flow rates can be obtained from the ideal gas law:

$$\dot{n} = \frac{P\dot{V}}{RT}$$

The volumetric flow rate, temperature and pressure of the exhaust gases exiting the fuel cell can be entered into this equation to determine the molar flow rate:

$$\dot{n} = \frac{(101.3 \text{ kPa})\left(130 \frac{m^3}{hr}\right)}{\left(8.314\times10^{-3} \frac{kJ}{mol\cdot K}\right)(1023 \text{ K})} = \_\_\_\_\_\_ \frac{mol}{hr}$$

If we multiply this molar flow rate by the molar fraction of its components, the molar flow rates $\dot{n}_1$, $\dot{n}_2$, $\dot{n}_3$ and $\dot{n}_4$ can be obtained as shown in the following equations. For the dry air, the composition was considered to be 21 mol % oxygen and 79 mol % nitrogen.

$$\dot{n}_1 = y_1\dot{n} = \left(\_\_\_\_ \frac{mol\ air}{mol}\right)\left(0.21 \frac{mol\ O_2}{mol\ air}\right)\left(\_\_\_\_\_\_ \frac{mol}{hr}\right)$$

$$\dot{n}_1 = \_\_\_\_\_ \frac{mol\ O_2}{hr}$$

$$\dot{n}_2 = y_2\dot{n} = \left(\_\_\_\_\_ \frac{mol\ air}{mol}\right)\left(\_\_\_\_\_ \frac{mol\ N_2}{mol\ air}\right)\left(\_\_\_\_\_\_\_ \frac{mol}{hr}\right)$$

$$\dot{n}_2 = \_\_\_\_\_\_\_ \frac{mol\ N_2}{hr}$$

$$\dot{n}_3 = y_3\dot{n} = \left(\_\_\_\_\_\_ \frac{\text{mol H}_2\text{O}}{\text{mol}}\right)\left(\_\_\_\_\_\_\_ \frac{\text{mol}}{\text{hr}}\right)$$

$$\dot{n}_3 = \_\_\_\_\_\_\_ \frac{\text{mol H}_2\text{O}}{\text{hr}}$$

To determine the molar flow rate of the saturated water, we can use the ideal gas law. It is noted that the temperature is not given directly in the diagram but the pressure value of 2177 kPa can be used to look for a temperature value in the saturated steam tables. Hence, the temperature is found to be:

T = _________ °C

Substituting this value into the ideal gas equation, yields:

$$\dot{n}_4 = \frac{P\dot{V}_4}{RT} = \frac{(\_\_\_\_\_\_\_ \text{kPa})\left(0.7\frac{\text{m}^3}{\text{hr}}\right)}{\left(8.314\times10^{-3}\frac{\text{kJ}}{\text{mol}\cdot\text{K}}\right)(\_\_\_\_\_\_\_ \text{K})}$$

$$\dot{n}_4 = \_\_\_\_\_\_\_ \frac{\text{mol H}_2\text{O}}{\text{hr}}$$

Since there is no other stream mixing with the gases coming from the fuel cell, their flow rates will remain constant:

$$\dot{n}_1 = \dot{n}_5 = \_\_\_\_\_ \frac{\text{mol H}_2\text{O}}{\text{hr}}$$

$$\dot{n}_2 = \dot{n}_6 = \_\_\_\_\_\_\_ \frac{\text{mol H}_2\text{O}}{\text{hr}}$$

$$\dot{n}_3 = \dot{n}_7 = \_\_\_\_\_\_\_ \frac{\text{mol H}_2\text{O}}{\text{hr}}$$

In addition, since there is no accumulation of water in the system, the molar flow rate of the liquid water entering the boiler will be the same as the flow rate of the saturated steam produced.

$$\dot{n}_4 = \dot{n}_8 = \_\_\_\_\_\_\_ \frac{\text{mol H}_2\text{O}}{\text{hr}}$$

Recalling, the reference states for both the boiling water and the exhaust gases from a fuel cell are:

Air @ T = 750 °C, P = 101.3 kPa
Liquid water @ Triple point (0°C)

The enthalpy of saturated liquid water at the triple point is equal to zero. This will help simplify the determination of enthalpies of saturated water by simply looking for the values in the steam tables. Thus, the enthalpies of saturated water were found in Table B.6 of Felder & Rousseau to be:

$$\hat{H}_4 = 1.886 \frac{\text{kJ}}{\text{mol}} \text{(saturated liquid water @ T = 25 °C)}$$

$$\hat{H}_8 = \underline{\hspace{3em}} \frac{\text{kJ}}{\text{mol}} \text{(saturated steam @ P = 2177 kPa)}$$

The enthalpy of the superheated steam coming from the fuel cell can be also determined by looking the value in Table B.7 of Felder & Rousseau, at a pressure of 101.3 kPa and a temperature of 750 °C.

$$\hat{H}_3 = \underline{\hspace{3em}} \frac{\text{kJ}}{\text{mol}} \text{(superheated steam @ P = 101.3 kPa, T = 750 °C)}$$

In addition, it can be seen that the selected reference state for the air is at the same conditions present in the input stream. Hence, the enthalpy of the dry air entering the boiler is equal to zero.

With these considerations, the updated table for the boiling process will look as follows:

Substance	$\dot{n}_{in}, \frac{\textbf{mol}}{\textbf{hr}}$	$\hat{H}_{in}, \frac{\textbf{kJ}}{\textbf{mol}}$	$\dot{n}_{out}, \frac{\textbf{mol}}{\textbf{hr}}$	$\hat{H}_{out}, \frac{\textbf{kJ}}{\textbf{mol}}$
O_2	270	0	270	$\hat{H}_5$
N_2	_______	0	_______	$\hat{H}_6$
H_2O (superheated steam from SOFC)	263.2	_______	263.2	$\hat{H}_7$
H_2O (saturated)	_______	1.886	_______	________

The definition of enthalpy as a function of temperature is given by the following equation:

$$\dot{H}_i = \int_{T_{ref}}^{T} C_{p,i}(T)dT$$

where:

i represents each one of the species entering or leaving the boiler.

T_{ref} = Temperature of the species *i* at the reference state.

T = Temperature of the species *i* at the actual process conditions.

The equations for the heat capacity at constant pressure for the species in this problem can be obtained from Table B.2 of Felder & Rousseau, and are shown below:

$$C_{p,O_2(g)}\left(\frac{\text{kJ}}{\text{mol}\cdot{}^\circ\text{C}}\right)=(\ \underline{\qquad\qquad}\ )+\left(1.158\times10^{-5}\right)T-(\ \underline{\qquad\qquad}\ )T^2+\left(1.311\times10^{-12}\right)T^3$$

$$C_{p,N_2(g)}\left(\frac{\text{kJ}}{\text{mol}\cdot{}^\circ\text{C}}\right)=\left(2.9\times10^{-2}\right)+(\ \underline{\qquad\qquad}\ )T+(\ \underline{\qquad\qquad}\ )T^2-(\ \underline{\qquad\qquad}\ )T^3$$

$$C_{p,H_2O(l)}\left(\frac{\text{kJ}}{\text{mol}\cdot{}^\circ\text{C}}\right)=\underline{\qquad\qquad\qquad}$$

$$C_{p,H_2O(g)}\left(\frac{\text{kJ}}{\text{mol}\cdot{}^\circ\text{C}}\right)=(\ \underline{\qquad\qquad}\ )+(\ \underline{\qquad\qquad}\ )T+(\ \underline{\qquad\qquad}\ )T^2-(\ \underline{\qquad\qquad}\ )T^3$$

Since there is no phase change for the air, the enthalpy can be obtained by integrating the heat capacity equation with respect to temperature. The temperature interval will be from the reference temperature to the output temperature.

$$\hat{H}_5=\int_{750^\circ C}^{T}C_{p,\,O_2}(T)dT$$

$$\hat{H}_6=\int_{750^\circ C}^{T}C_{p,\,N_2}(T)dT$$

Substituting and integrating the heat capacity equations into the definition of the enthalpies $\hat{H}_5,\ \hat{H}_6$, we get:

$$\hat{H}_5=(\ \underline{\qquad\qquad}\ )T+\left(5.79\times10^{-6}\right)T^2-(\ \underline{\qquad\qquad}\ )T^3+\left(3.278\times10^{-13}\right)T^4\Big|_{750^\circ C}^{T}$$

$$\hat{H}_6=\left(2.9\times10^{-2}\right)T+(\ \underline{\qquad\qquad}\ )T^2+(\ \underline{\qquad\qquad}\ )T^3-(\ \underline{\qquad\qquad}\ )T^4\Big|_{750^\circ C}^{T}$$

Entering the temperature values into this equation yields:

$$\hat{H}_5 = (\ \_\_\_\_\_\_\_\_\ )[T - \_\_\_\_\ ^\circ C] + (5.79\times10^{-6})\left[T^2 - (\_\_\_\_\ ^\circ C)^2\right]$$
$$-(\ \_\_\_\_\_\_\_\_\ )\left[T^3 - (\_\_\_\_\_\ ^\circ C)^3\right] + (3.278\times10^{-13})\left[T^4 - (\_\_\_\_\_\ ^\circ C)^4\right]$$

$$\hat{H}_5 = \left[(3.278\times10^{-13})T^4 - (\ \_\_\_\_\_\_\_\_\_\ )T^3 + (5.79\times10^{-6})T^2 + (\ \_\_\_\_\_\_\_\_\_\ )T - 24.332\right]\frac{kJ}{mol}$$

$$\hat{H}_6 = (2.9\times10^{-2})[T - \_\_\_\_\ ^\circ C] + (\ \_\_\_\_\_\_\_\_\_\ )\left[T^2 - (\_\_\_\_\ ^\circ C)^2\right]$$
$$+(\ \_\_\_\_\_\_\_\_\_\ )\left[T^3 - (\_\_\_\_\ ^\circ C)^3\right] - (\ \_\_\_\_\_\_\_\_\_\ )\left[T^4 - (\_\_\_\_\ ^\circ C)^4\right]$$

$$\hat{H}_6\left(\frac{kJ}{mol}\right) = -(\ \_\_\_\_\_\_\_\_\_\ )T^4 + (\ \_\_\_\_\_\_\_\_\_\ )T^3 + (\ \_\_\_\_\_\_\_\_\_\ )T^2 + (2.9\times10^{-2})T$$
$$-\_\_\_\_\_\_\_\_\_\_$$

The only remaining enthalpy is for the steam exiting in the air/steam mixture. In order to calculate this, a hypothetical path will be constructed as follows:

$$\hat{H}_7 = \hat{H}_{7a}(\text{water at reference conditions}) + \hat{H}_{7b}(\text{liquid water from ref. conditions to boiling point})$$
$$+\hat{H}_{7c}(\text{heat of vaporization}) + \hat{H}_{7d}(\text{steam from boiling point to process temperature})$$

The enthalpy of liquid water at the triple point of water is equal to zero. Hence, the first term of this equation can be eliminated.

Each one of the remaining enthalpies in this equation are defined as follows:

$$\hat{H}_{7b} = \int_{T_{ref}}^{T_{bp}} C_{p,H_2O(l)}(T)dT$$
$$\hat{H}_{7c} = \Delta H_v = \_\_\_\_\_\_\_\_\frac{kJ}{mol}\ \text{(obtained from Table B.1 of Felder \& Rousseau)}$$
$$\hat{H}_{7d} = \int_{T_{bp}}^{T} C_{p,H_2O(v)}(T)dT$$

The heat capacity equations of liquid and vapor water can be substituted into the second and fourth terms of this equations, respectively to yield:

$$\hat{H}_{7b} = \int_{0^\circ C}^{100^\circ C} 7.54\times10^{-2}dT$$

$$\hat{H}_{7d} = \int_{100^\circ C}^{T} (\ \_\_\_\_\_\_\_\_\_\ ) + (\ \_\_\_\_\_\_\_\_\_\ )T + (\ \_\_\_\_\_\_\_\_\_\ )T^2 - (\ \_\_\_\_\_\_\_\_\_\ )T^3 dT$$

Integrating these two equations and substituting the temperature values, we get:

$$\hat{H}_{7b} = \left\{\left(7.54\times10^{-2}\right)[\_\_\_\_ °C - \_\_\_\_ °C]\right\}\frac{kJ}{mol}$$

$$\hat{H}_{7b} = \_\_\_\_\_\_\_ \frac{kJ}{mol}$$

$$\hat{H}_{7d}\left(\frac{kJ}{mol}\right) = (\ \_\_\_\_\_\_\_\_\_\ )[T - \_\_\_\_\_ °C] + (\ \_\_\_\_\_\_\_\_\_\ )\left[T^2 - (\_\_\_\_\_ °C)^2\right]$$
$$+ (\ \_\_\_\_\_\_\_\_\_\ )\left[T^3 - (\_\_\_\_\_ °C)^3\right] - (\ \_\_\_\_\_\_\_\_\_\ )\left[T^4 - (\_\_\_\_\_ °C)^4\right]$$

$$\hat{H}_{7d}\left(\frac{kJ}{mol}\right) = -(\ \_\_\_\_\_\_\_\_\_\ )T^4 + (\ \_\_\_\_\_\_\_\_\_\ )T^3 + (\ \_\_\_\_\_\_\_\_\_\ )T^2$$
$$+ (\ \_\_\_\_\_\_\_\_\_\ )T - 3.383$$

The enthalpy of the superheated water exiting the boiler can now be calculated by adding the enthalpies $\hat{H}_{7b}$, $\hat{H}_{7c}$, and $\hat{H}_{7d}$:

$$\hat{H}_7 = \hat{H}_{7b} + \hat{H}_{7c} + \hat{H}_{7d}$$

$$\hat{H}_7 = \_\_\_\_\_\_ \frac{kJ}{mol} + 40.656\frac{kJ}{mol} + \left[-(\ \_\_\_\_\_\_\_\_\_\ )T^4 + (\ \_\_\_\_\_\_\_\_\_\ )T^3 + (\ \_\_\_\_\_\_\_\_\_\ )T^2\right.$$
$$\left. + (\ \_\_\_\_\_\_\_\_\_\ )T - 3.383\right]\frac{kJ}{mol}$$

$$\hat{H}_7\left(\frac{kJ}{mol}\right) = -(\ \_\_\_\_\_\_\_\_\_\ )T^4 + (\ \_\_\_\_\_\_\_\_\_\ )T^3 + (\ \_\_\_\_\_\_\_\_\_\ )T^2$$
$$+ (\ \_\_\_\_\_\_\_\_\_\ )T + 44.813$$

So far, we have found expressions for the some of the unknown enthalpies as a function of the temperature of the air/steam mixture exiting the boiler, T. In order to determine the enthalpy values, we need to determine this temperature by using the energy balance equation.

Substituting the enthalpy and flow rate of each component into the energy balance equation will yield:

$$\Delta\dot{H} = \dot{n}_5\hat{H}_5 + \dot{n}_6\hat{H}_6 + \dot{n}_7\hat{H}_7 + \dot{n}_8\hat{H}_8 - \dot{n}_3\hat{H}_3 - \dot{n}_4\hat{H}_4 = 0$$

$$\begin{aligned}\Delta\dot{H} = 270\frac{\text{mol}}{\text{hr}}\Big[\big(\underline{\qquad\qquad}\big)T^4 - \left(2.025\times10^{-9}\right)T^3 + \big(\underline{\qquad\qquad}\big)T^2 + \left(2.91\times10^{-2}\right)T\\ -24.332\Big]\frac{\text{kJ}}{\text{mol}} + \underline{\qquad}\frac{\text{mol}}{\text{hr}}\Big[-\left(7.178\times10^{-13}\right)T^4 + \big(\underline{\qquad\qquad}\big)T^3 + \big(\underline{\qquad\qquad}\big)T^2\\ +\big(\underline{\qquad\qquad}\big)T - 22.947\Big]\frac{\text{kJ}}{\text{mol}} + 263.2\frac{\text{mol}}{\text{hr}}\Big[-\big(\underline{\qquad\qquad}\big)T^4 + \left(2.535\times10^{-9}\right)T^3\\ +\big(\underline{\qquad\qquad}\big)T^2 + \left(3.346\times10^{-2}\right)T + \underline{\qquad\quad}\Big]\frac{\text{kJ}}{\text{mol}} + \underline{\qquad}\frac{\text{mol}}{\text{hr}}\left(50.384\frac{\text{kJ}}{\text{mol}}\right)\\ -263.2\frac{\text{mol}}{\text{hr}}\left(\underline{\qquad\quad}\frac{\text{kJ}}{\text{mol}}\right) - \underline{\qquad\quad}\frac{\text{mol}}{\text{hr}}\left(1.886\frac{\text{kJ}}{\text{mol}}\right) = 0\end{aligned}$$

After multiplying the molar flow rates by each enthalpy equation and adding similar terms, we have:

$$\begin{aligned}\Delta\dot{H} = \Big[-\left(8.766\times10^{-10}\right)T^4 + \big(\underline{\qquad\qquad}\big)T^3 + \big(\underline{\qquad\qquad}\big)T^2 + 46.105T\\ -\underline{\qquad\quad}\Big]\frac{\text{kJ}}{\text{hr}} = 0\end{aligned}$$

We can solve this equation by using computer software or numerically to find the temperature value to be:

$$\boxed{T \approx \underline{\quad\quad}\ ^{\circ}\text{C}}$$

b) Determine the enthalpy of the air/steam mixture exiting the boiler:

Strategy

The equations for the enthalpy of each substance as a function of temperature obtained in part a) of this problem can be used to obtain the enthalpy of the mixture.

Solution

The enthalpy of the humid air exiting the boiler can be obtained by multiplying the molar fraction of each component by its corresponding enthalpy. This can be written as:

$$\hat{H}_m = y_5\hat{H}_5 + y_6\hat{H}_6 + y_7\hat{H}_7$$

The enthalpies $\hat{H}_5$, $\hat{H}_6$ and $\hat{H}_7$ can be calculated by entering the temperature value found in part a) into the corresponding enthalpy equation:

$$\hat{H}_5 = \left[\left(3.278\times10^{-13}\right)(\_\_\_\_\_\ °\text{C})^4 - (\_\_\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\ °\text{C})^3 + \left(5.79\times10^{-6}\right)(\_\_\_\_\_\ °\text{C})^2 + (\_\_\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\ °\text{C}) - 24.332\right]\frac{\text{kJ}}{\text{mol}}$$

$$\hat{H}_5 = \_\_\_\_\_\_\_\_\_\ \frac{\text{kJ}}{\text{mol}}$$

$$\hat{H}_6 = \left[-(\_\_\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\ °\text{C})^4 + (\_\_\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\ °\text{C})^3 + (\_\_\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\ °\text{C})^2 + \left(2.9\times10^{-2}\right)(\_\_\_\_\_\ °\text{C}) - \_\_\_\_\_\_\_\_\_\right]\frac{\text{kJ}}{\text{mol}}$$

$$\hat{H}_6 = -11.067\frac{\text{kJ}}{\text{mol}}$$

$$\hat{H}_7 = \left[-(\_\_\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\ °\text{C})^4 + (\_\_\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\ °\text{C})^3 + \left(3.44\times10^{-6}\right)(\_\_\_\_\_\ °\text{C})^2 + (\_\_\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\ °\text{C}) + 44.813\right]\frac{\text{kJ}}{\text{mol}}$$

$$\hat{H}_7 = 58.887\frac{\text{kJ}}{\text{mol}}$$

Substituting the individual enthalpies and the molar fractions into the equation for the enthalpy of the mixture, we get:

$$\hat{H}_m = \left(0.17\frac{\text{mol H}_2\text{O}}{\text{mol}}\right)\left(58.887\frac{\text{kJ}}{\text{mol H}_2\text{O}}\right) + \left(0.83\frac{\text{mol air}}{\text{mol}}\right)\left(\_\_\_\_\_\ \frac{\text{mol O}_2}{\text{mol air}}\right)\left(\_\_\_\_\_\_\_\_\_\ \frac{\text{kJ}}{\text{mol O}_2}\right) + \left(\_\_\_\_\ \frac{\text{mol air}}{\text{mol}}\right)\left(\_\_\_\_\ \frac{\text{mol N}_2}{\text{mol air}}\right)\left(-11.067\frac{\text{kJ}}{\text{mol N}_2}\right)$$

$$\boxed{\hat{H}_m = \_\_\_\_\_\_\_\_\ \frac{\text{kJ}}{\text{mol}}}$$