Chapter 7   Name: _____
Energy and Energy Balances   Date: _____

In Chapter 7, it will be shown how energy balances can be used to solve problems involving chemical processes. Different types of problem including the types of energy and how it is transferred in and out of a system, which can be either open or closed. The following problems build upon the fundamentals covered in your text as applied to hydrogen processing, hydrogen as an energy carrier, and the use of hydrogen in fuel cells.

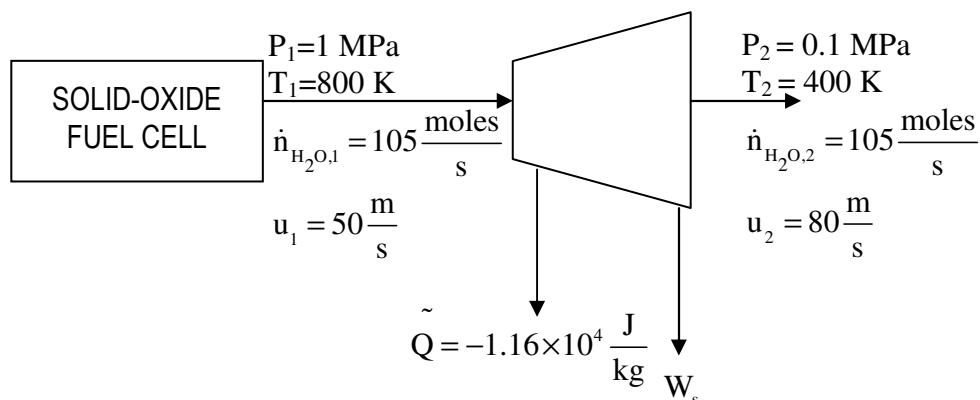7.4-2 Energy Balance on a Turbine
7.5-1 Use of Tabulated Enthalpy Data
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Example 7.4-2 Energy Balance on a Turbine

A system combining a solid-oxide fuel cell with a gas turbine has been proved to achieve higher operating efficiencies at high pressures. The exhaust gases from the fuel cell are entering a steam turbine as shown in the following figure.

Determine the shaft work done by the turbine.

Strategy

This problem can be solved by performing energy balances on the turbine, using the information given in the process diagram to obtain the different quantities involved in the energy balance.

Solution

The amount of energy transferred to or from a system as heat or shaft work is equal to the rates of change of the enthalpy, and the potential and kinetic energies. This definition can be written as follows:

$$\Delta \dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p = \dot{Q} - \dot{W}_s$$

The rate of change of enthalpy is given by:

$$\Delta \dot{H} = \dot{n} \left(\hat{H}_2 - \hat{H}_1 \right)$$

The enthalpies of the steam entering and exiting the turbine can be found in Table 2-305 of *Perry's Chemical Engineers' Handbook*, 8th Edition.

P(MPa)	T(K)	$\hat{H}$ (kJ/mol)
1	800	63.729
0.1	400	_____

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Substituting the values from the table into the equation for the rate of change of enthalpy gives:

$$\Delta\dot{H} = 105\frac{\text{moles}}{\text{s}}\left(\underline{\quad\quad}\frac{\text{kJ}}{\text{mol}} - 63.729\frac{\text{kJ}}{\text{mol}}\right)$$

$$\Delta\dot{H} = -\underline{\quad\quad\quad}\text{ kW}$$

Since there is a difference in the velocity of the steam between the inlet and outlet streams, the difference in the kinetic energy must be obtained as follows:

$$\Delta\dot{E}_k = \frac{\dot{m}}{2}\left(u_2^2 - u_1^2\right)$$

Entering the mass flow rate and the velocities into this equation yields:

$$\Delta\dot{E}_k = \frac{\underline{\quad}\frac{\text{moles}}{\text{s}}\left(\frac{1\text{ kmol}}{1000\text{ moles}}\right)\left(\frac{18\text{ kg}}{\text{kmol}}\right)}{2}\left[\left(80\frac{\text{m}}{\text{s}}\right)^2 - \left(\underline{\quad\quad}\frac{\text{m}}{\text{s}}\right)^2\right]\left(\frac{1\text{ kJ}}{\underline{\quad\quad}\text{ J}}\right)$$

$$\Delta\dot{E}_k = \underline{\quad\quad\quad}\text{ kW}$$

The potential energy is due to a difference in the height. Since both inlet and outlet streams are at the same level, the term for the potential energy in the energy balance can be neglected.

The only term that remains to be calculated is the heat loss term $\dot{Q}$, which can be obtained by multiplying the mass flow rate by the heat loss in $\frac{\text{J}}{\text{kg}}$:

$$\dot{Q} = \dot{m}\tilde{Q}$$

By substituting the information from the process diagram we get:

$$\dot{Q} = \underline{\quad\quad}\frac{\text{moles}}{\text{s}}\left(\frac{1\text{ kmol}}{1000\text{ moles}}\right)\left(\frac{18\text{ kg}}{1\text{ kmol}}\right)\left(\underline{\quad\quad}\frac{\text{J}}{\text{kg}}\right)\left(\frac{1\text{ kJ}}{1000\text{ J}}\right)$$

$$\dot{Q} = \underline{\quad\quad}\text{ kW}$$

Now that all the terms in the energy balance are known, we can substitute these quantities and solve for the shaft work $\dot{W}_s$:

$$\dot{W}_s = \dot{Q} - \Delta\dot{E}_k - \Delta\dot{E}_p - \Delta\dot{H} = -\underline{\quad\quad\quad}\text{ kW} - 3.69\text{ kW} - \left(\underline{\quad\quad\quad}\text{ kW}\right)$$

$$\boxed{\dot{W}_s = 1501.39\text{ kW}}$$

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7.5-1 Use of Tabulated Enthalpy Data

Methanol is the fuel used for producing electricity in direct-methanol fuel cells. The following table is showing thermodynamic properties of this fuel. This data was obtained from Table 2-234 of *Perry's Chemical Engineers' Handbook*, 8th Edition.

Phase	T, K	P, kPa	$\hat{V}, \dfrac{L}{mol}$	$\hat{H}, \dfrac{kJ}{mol}$
Liquid	285	8.2787	4.0125×10^{-2}	-4.4347
Liquid	315	38.692	4.1595×10^{-2}	-1.9842
Vapor	330	74.453	35.164	35.060
Vapor	345	134.47	20.052	34.485

Calculate the changes in enthalpy and internal energy from 285 K to 315 K, and from 330 K to 345 K.

Strategy

We need to use the values from the table and the definition of internal energy to find the solution to this problem.

Solution

The change in the enthalpy will be given by the following equation:

$$\Delta \hat{H} = \hat{H}_f - \hat{H}_i$$

where:

$\hat{H}_f = $ Enthalpy of methanol at the final temperature

$\hat{H}_i = $ Enthalpy of methanol at the initial temperature

First we will calculate the change in enthalpy from 285 K to 315 K (liquid phase). Substituting the values from the table into the equation for the change in enthalpy, we have:

$$\Delta \hat{H}_{285\,K\,-\,315\,K} = \left(-1.9842 \frac{kJ}{mol} \right) - \left(\underline{\qquad} \frac{kJ}{mol} \right)$$

$$\boxed{\Delta \hat{H}_{285\,K\,-\,315\,K} = \underline{\qquad} \frac{kJ}{mol}}$$

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For the temperature change from 330 to 345 (vapor phase), the change in enthalpy can be calculated as follows:

$$\Delta \hat{H}_{330\,K\text{ - }345\,K} = \underline{\hspace{2cm}} \frac{kJ}{mol} - \underline{\hspace{2cm}} \frac{kJ}{mol}$$

$$\boxed{\Delta \hat{H}_{330\,K\text{ - }345\,K} = -0.575 \frac{kJ}{mol}}$$

The values we just obtained for the changes in enthalpy will be used for calculating the change in internal energy for methanol in both the liquid and vapor phases. The internal energy definition is given by the following equation:

$$\Delta \hat{U} = \Delta \hat{H} - \Delta(P\hat{V})$$

The term $\Delta(P\hat{V})$ can be rewritten as follows

$$\Delta(P\hat{V}) = (P_f \hat{V}_f - P_i \hat{V}_i)$$

Where the sub index f refers to the final condition, and the sub index i to the initial condition of methanol. Inserting this equation into the equation for $\Delta \hat{U}$ yields:

$$\Delta \hat{U} = \Delta \hat{H} - (P_f \hat{V}_f - P_i \hat{V}_i)$$

We can insert the quantities from the table to determine the change in internal energy for the liquid phase when increasing its temperature from 285 K to 315 K.

$$\Delta \hat{U}_{285K-315K} = \underline{\hspace{1.5cm}} \frac{kJ}{mol} - \underline{\hspace{1.5cm}} kPa \left(4.1595 \text{ x } 10^{-2} \frac{L}{mol} \right)\left(\frac{1\,m^3}{1000\,L} \right)$$

$$+ 8.2787 \text{ kPa} \left(\underline{\hspace{1.5cm}} \frac{L}{mol} \right)\left(\frac{1\,m^3}{1000\,L} \right)$$

$$\boxed{\Delta \hat{U}_{285K-315K} = \underline{\hspace{1.5cm}} \frac{kJ}{mol}}$$

The same procedure is followed to difference in the internal energy of methanol in the vapor phase from 330 to 345 K.

$$\Delta \hat{U} = -0.575 \frac{kJ}{mol} - \left[134.47 \text{ kPa} \left(\underline{\hspace{1cm}} \frac{L}{mol} \right) - \underline{\hspace{1cm}} kPa \left(35.164 \frac{L}{mol} \right) \right]\left(\frac{1\,m^3}{1000\,L} \right)$$

$$\boxed{\Delta \hat{U}_{330K-345K} = -0.6533 \frac{kJ}{mol}}$$

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7.6-1 Energy Balance on a One-Component Process

In a steam-methane reforming process for hydrogen production, water is fed to a boiler which will produce superheated steam. This steam is used to heat the reactants before entering the reactor tubes. A diagram of the process occurring in the boiler is shown below:

$$425 \ \frac{\text{kg H}_2\text{O}}{\text{hr}}$$

T = 30°C
P = 4.8 MPa

Superheated Steam
T = 360°C
P = 4.8 MPa
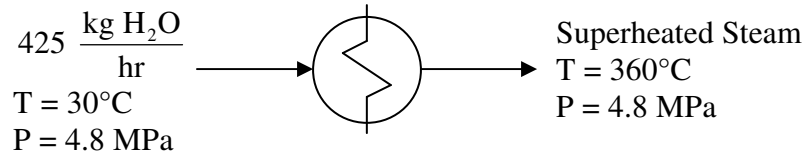
Strategy

The starting point for solving this problem will be the general energy balance equation. We will also need the enthalpies of both the liquid water entering the boiler and the steam produced.

Solution

We can start by applying the energy balance equation to this process:

$$\Delta \dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p = \dot{Q} - \dot{W}_s$$

For the process taking place in this problem, the following assumptions can be made:

$\Delta \dot{E}_k = 0$ Since there is no information given about the velocities of the inlet and outlet streams the velocity difference will be neglected

$\Delta \dot{E}_p = 0$ There is no considerable difference in height between the inlet and outlet streams

$\dot{W}_s = 0$ There are no mechanical parts moving

Thus, the energy balance equation will be described by:

$$\dot{Q} = \Delta \dot{H}$$

The change in the enthalpy $\Delta \dot{H}$ can be calculated as follows:

$$\Delta \dot{H} = \dot{n}(\hat{H}_{out} - \hat{H}_{in})$$

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The enthalpy of the liquid water entering the system can be obtained by interpolation using the data from Table 2-305 from *Perry's Chemical Engineers' Handbook*, 8th Edition, summarized in the following table.

	1 MPa	4.8 MPa	5 MPa
300 K	$\underline{\hspace{2cm}} \dfrac{kJ}{mol}$		$2.1106 \dfrac{kJ}{mol}$
303.15 K	$\hat{H}_{@\,T=303.15\,K,P=1\,MPa}$	$\hat{H}_{@\,T=303.15\,K,P=4.8\,MPa}$	$\hat{H}_{@\,T=303.15\,K,P=5\,MPa}$
400 K	$9.6106 \dfrac{kJ}{mol}$		$\underline{\hspace{2cm}} \dfrac{kJ}{mol}$

We can interpolate down at constant pressure, then across at constant temperature.

From the table shown above, we need to calculate first the values of the molar enthalpy at a constant temperature of 303.15 K at a pressure of 1 MPa. We setup the linear interpolation as:

$$\frac{T_{mid} - T_{low}}{T_{high} - T_{low}} = \frac{\hat{H}_{@\,T=303.15\,K,P=1\,MPa} - \hat{H}_{@\,T_{low}}}{\hat{H}_{@\,T_{high}} - \hat{H}_{@\,T_{low}}}$$

Entering the values from the table into this equation and solving for $\hat{H}_{@\,T=303.15\,K,P=1\,MPa}$ yields:

$$\hat{H}_{@\,T=303.15\,K,P=1\,MPa} = \left(\frac{303.15\,K - 300\,K}{400\,K - 300\,K}\right)\left(9.6106 \frac{kJ}{mol} - \underline{\hspace{1cm}} \frac{kJ}{mol}\right) + \underline{\hspace{1cm}} \frac{kJ}{mol}$$

$$\hat{H}_{@\,T=303.15\,K,P=1\,MPa} = \underline{\hspace{1.5cm}} \frac{kJ}{mol}$$

The same procedure is repeated at the temperature of 303.15 K, only that this time will be done for the pressure of 5 MPa.

$$\frac{T_{mid} - T_{low}}{T_{high} - T_{low}} = \frac{\hat{H}_{@\,T=303.15\,K,P=5\,MPa} - \hat{H}_{@\,T_{low}}}{\hat{H}_{@\,T_{high}} - \hat{H}_{@\,T_{low}}}$$

Entering the values from the table into this equation and solving for $\hat{H}_{@\,T=303.15\,K,P=5\,MPa}$ yields:

$$\hat{H}_{@\,T=303.15\,K,P=5\,MPa} = \left(\frac{303.15\,K - 300\,K}{400\,K - 300\,K}\right)\left(\underline{\hspace{1cm}} \frac{kJ}{mol} - 2.1106 \frac{kJ}{mol}\right) + 2.1106 \frac{kJ}{mol}$$

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$$\hat{H}_{@T=303.15 \text{ K},P=5 \text{ MPa}} = \underline{\qquad} \frac{\text{kJ}}{\text{mol}}$$

Now, the only remaining step to be done is to obtain the value of the molar enthalpy of liquid water at the temperature and pressure of the feed stream:

$$\frac{P_{mid} - P_{low}}{P_{high} - P_{low}} = \frac{\hat{H}_{@ \text{ T=303.15 K},P=4.8 \text{ MPa}} - \hat{H}_{@ P_{low}}}{\hat{H}_{@ P_{high}} - \hat{H}_{@ P_{low}}}$$

Substituting the values from the table into this equation, we can solve for $\hat{H}_{@T=303.15 \text{ K},P=4.8 \text{ MPa}}$ to get:

$$\hat{H}_{@T=303.15 \text{ K},P=4.8 \text{ MPa}} = \left(\frac{\underline{\qquad} \text{ MPa} - 1 \text{ MPa}}{5 \text{ MPa} - 1 \text{ MPa}} \right) \left(\underline{\qquad} \frac{\text{kJ}}{\text{mol}} - \underline{\qquad} \frac{\text{kJ}}{\text{mol}} \right)$$
$$+ \underline{\qquad} \frac{\text{kJ}}{\text{mol}}$$

$$\hat{H}_{@T=303.15 \text{ K},P=4.8 \text{ MPa}} = \underline{\qquad} \frac{\text{kJ}}{\text{mol}}$$

If we substitute the calculated enthalpies into the initial table, we get:

	1 MPa	4.8 MPa	5 MPa
300 K	$\underline{\qquad} \frac{\text{kJ}}{\text{mol}}$		$2.1106 \frac{\text{kJ}}{\text{mol}}$
303.15 K	$\underline{\qquad} \frac{\text{kJ}}{\text{mol}}$	$\underline{\qquad} \frac{\text{kJ}}{\text{mol}}$	$\underline{\qquad} \frac{\text{kJ}}{\text{mol}}$
400 K	$9.6106 \frac{\text{kJ}}{\text{mol}}$		$\underline{\qquad} \frac{\text{kJ}}{\text{mol}}$

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To determine the enthalpy of the superheated steam exiting this process, a similar procedure will be followed, only that the values for vapor water at T = 360 °C and P = 4.8 MPa will be determined through interpolation. The following table is showing the enthalpy values for the steam:

	1 MPa	4.8 MPa	5 MPa
600 K	$51.123 \ \dfrac{kJ}{mol}$		$49.734 \ \dfrac{kJ}{mol}$
633.15 K	$\hat{H}_{@T=633.15 \ K,P=1 \ MPa}$	$\hat{H}_{@T=633.15 \ K,P=4.8 \ MPa}$	$\hat{H}_{@T=633.15 \ K,P=5 \ MPa}$
700 K	$54.087 \ \dfrac{kJ}{mol}$		$53.286 \ \dfrac{kJ}{mol}$

After doing the interpolation procedure and entering the values into the table, we get:

	1 MPa	4.8 MPa	5 MPa
600 K	$51.123 \ \dfrac{kJ}{mol}$		$49.734 \ \dfrac{kJ}{mol}$
633.15 K	$\underline{\hspace{2cm}} \ \dfrac{kJ}{mol}$	$\underline{\hspace{2cm}} \ \dfrac{kJ}{mol}$	$\underline{\hspace{2cm}} \ \dfrac{kJ}{mol}$
700 K	$54.087 \ \dfrac{kJ}{mol}$		$53.286 \ \dfrac{kJ}{mol}$

Now we can determine the heat required to boil the feed water, by calculating the enthalpy change:

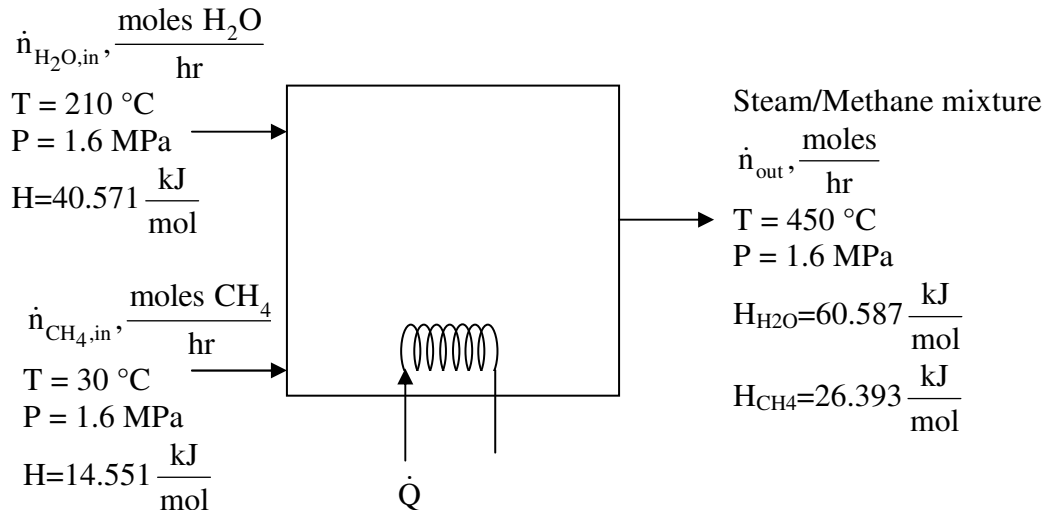$$\dot{Q} = \Delta \dot{H} = \dot{n}(\hat{H}_{out} - \hat{H}_{in})$$

Entering the corresponding quantities into this equation gives:

$$\dot{Q} = \Delta \dot{H} = \left(425 \frac{kg \ H_2O}{hr} \right)\left(\frac{1 \ kmol \ H_2O}{18 \ kg \ H_2O} \right)\left(\frac{1000 \ mol \ H_2O}{1 \ kmol \ H_2O} \right)\left(\frac{1 \ hr}{\underline{\hspace{1cm}} \ s} \right)\left(\underline{\hspace{1cm}} \frac{kJ}{mol} - \underline{\hspace{1cm}} \frac{kJ}{mol} \right)$$

$$\boxed{\dot{Q} = \underline{\hspace{2cm}} \ kW}$$

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7.6-2 Energy Balance on a Two-Component Process

Hydrogen can be produced by steam-methane reforming. The feed to this process is heated to a temperature of 450 °C before entering the reaction chamber. What is the required flow rate of each gas if the heater consumes 2.5 kW of power to bring the reaction gases to the operation conditions? The feed to the reaction chamber contains a steam/methane molar ratio of 3.

$\dot{n}_{H_2O,in}$, $\dfrac{\text{moles } H_2O}{\text{hr}}$

T = 210 °C
P = 1.6 MPa
H=40.571 $\dfrac{kJ}{mol}$

$\dot{n}_{CH_4,in}$, $\dfrac{\text{moles } CH_4}{\text{hr}}$

T = 30 °C
P = 1.6 MPa
H=14.551 $\dfrac{kJ}{mol}$

$\dot{Q}$

Steam/Methane mixture

$\dot{n}_{out}$, $\dfrac{\text{moles}}{\text{hr}}$

T = 450 °C
P = 1.6 MPa

H_{H2O}=60.587 $\dfrac{kJ}{mol}$

H_{CH4}=26.393 $\dfrac{kJ}{mol}$

Strategy

We need to determine the unknown parameters by performing an energy balance on the heater and use the information about the steam/methane ratio given in the problem statement.

Solution

The energy equation is given by:

$$\Delta \dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p = \dot{Q} - \dot{W}_s$$

The changes in the kinetic and potential energies will be negligible since there are no significant differences in the velocity and the height between the inlet and outlet streams. Since there are no mechanical parts moving in the heater, the shaft work will be also neglected. Thus, the equation is reduced to:

$$\dot{Q} = \Delta \dot{H}$$

The change in enthalpy for this process will be given by:

$$\Delta \dot{H} = \dot{n}_{out} \hat{H}_{mix} - (\dot{n}_{H_2O, in} \hat{H}_{H_2O,in} + \dot{n}_{CH_4, in} \hat{H}_{CH_4,in})$$

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Substituting this equation into the energy balance equation yields:

$$\dot{Q} = \dot{n}_{out}\hat{H}_{mix} - (\dot{n}_{H_2O,\ in}\hat{H}_{H_2O,in} + \dot{n}_{CH_4,\ in}\hat{H}_{CH_4,in})$$

For the steam-methane reforming process, the ratio of the number of moles of steam to the number of moles of methane is equal to 3. This can be described by the following equation:

$$\dot{n}_{H_2O,\ in} = 3\dot{n}_{CH_4,\ in}$$

This equation can be used to determine the molar flow rate of the mixed gases exiting the heater:

$$\dot{n}_{out} = \dot{n}_{CH_4,\ in} + \dot{n}_{H_2O,\ in} = \dot{n}_{CH_4,\ in} + 3\dot{n}_{CH_4,\ in}$$

Substituting the last two equations in the general energy equation applied for this problem gives:

$$\dot{Q} = 4\dot{n}_{CH_4,\ in}\hat{H}_{mix} - (\ \underline{\hspace{2cm}}\ \hat{H}_{H_2O,in} + \dot{n}_{CH_4,\ in}\hat{H}_{CH_4,in})$$

Another quantity that remains unknown in this equation is the enthalpy of the mixture exiting the heater $\hat{H}_{mix}$, which can be calculated as follows:

$$\hat{H}_{mix} = y_{CH_4}\hat{H}_{CH_4} + y_{H_2O}\hat{H}_{H_2O}$$

Since the only components present in the outlet stream are water and methane, and the ratio of moles of steam to moles of methane is known, the molar fractions can be obtained as follows:

$$y_{CH_4} + y_{H_2O} = 1$$
$$y_{CH_4} + 3y_{CH_4} = 1$$

Solving for the molar fraction of methane y_{CH4}, we get:

$$y_{CH_4} = \underline{\hspace{2cm}}\frac{moles\ CH_4}{mol}$$

and the molar fraction of water can be obtained by substituting the molar fraction into the equation for the sum of the molar fractions:

$$y_{H_2O} = 1 - y_{CH_4} = 1 - \underline{\hspace{2cm}}$$

$$y_{H_2O} = \underline{\hspace{2cm}} \frac{\text{moles } H_2O}{\text{mol}}$$

Now the enthalpy of the mixture can be calculated by substituting the molar fractions and enthalpies of methane and water to yield:

$$\hat{H}_{mix} = y_{CH_4}\hat{H}_{CH_4} + y_{H_2O}\hat{H}_{H_2O} = \underline{\hspace{1cm}}\left(26.393\frac{kJ}{mol}\right) + \underline{\hspace{1cm}}\left(\underline{\hspace{1cm}}\frac{kJ}{mol}\right)$$

$$\hat{H}_{mix} = \underline{\hspace{1.5cm}}\frac{kJ}{mol}$$

Substituting the known enthalpies and the consumed power into the energy balance equation allows us to solve for the molar flow rate of methane being fed to the heater:

$$2.5 \text{ kW} = 4\dot{n}_{CH_4,in}\left(\underline{\hspace{1.5cm}}\frac{kJ}{mol}\right) - \left[3\dot{n}_{CH_4,in}\left(\underline{\hspace{1.5cm}}\frac{kJ}{mol}\right) + \dot{n}_{CH_4,in}\left(\underline{\hspace{1.5cm}}\frac{kJ}{mol}\right)\right]$$

$$\dot{n}_{CH_4,in} = \frac{2.5 \text{ kW}}{\left[4\left(\underline{\hspace{1.5cm}}\frac{kJ}{mol}\right) - 3\left(\underline{\hspace{1.5cm}}\frac{kJ}{mol}\right) - \underline{\hspace{1.5cm}}\frac{kJ}{mol}\right]}$$

$$\dot{n}_{CH_4,in} = \underline{\hspace{1.5cm}}\frac{\text{mol}}{s}\left(\frac{3600 \text{ s}}{1 \text{ hr}}\right)$$

$$\boxed{\dot{n}_{CH_4,in} = \underline{\hspace{1.5cm}}\frac{\text{mol}}{\text{hr}}}$$

Recalling the molar ratio of the components in the process:

$$\dot{n}_{H_2O,in} = 3\dot{n}_{CH_4,in}$$

Entering the molar flow rate of methane into this equation gives:
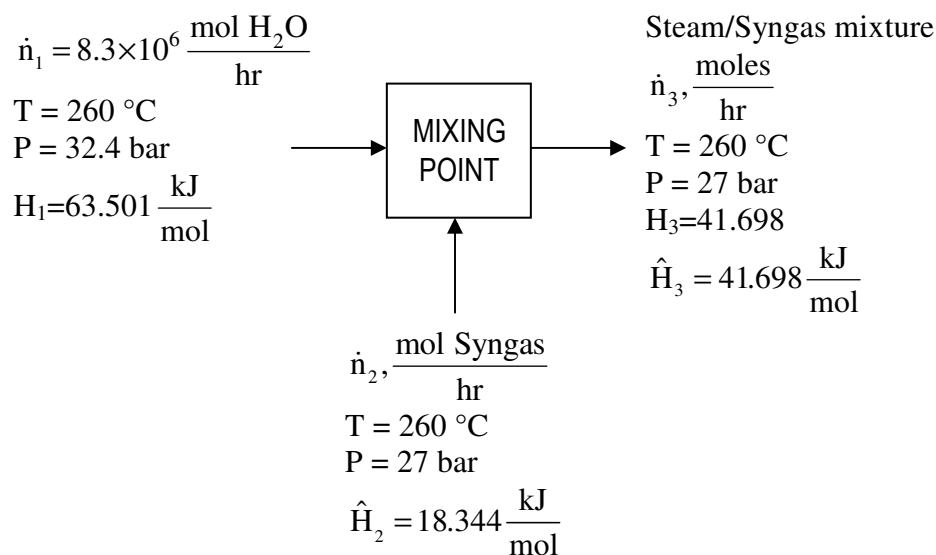
$$\dot{n}_{H_2O,in} = 3\left(\underline{\hspace{1.5cm}}\frac{\text{mol}}{\text{hr}}\right)$$

$$\boxed{\dot{n}_{H_2O,in} = \underline{\hspace{1.5cm}}\frac{\text{mol}}{\text{hr}}}$$

7.6-3 Simultaneous Material and Energy Balances

Synthesis gas is produced in a central-scale coal gasification process at a pressure of 27 bar and a temperature of 260 °F. This syngas stream will be mixed with steam before entering a shift reactor to further produce hydrogen. A diagram of this process is shown in the figure:

$$\dot{n}_1 = 8.3 \times 10^6 \frac{mol\ H_2O}{hr}$$

T = 260 °C
P = 32.4 bar

$$H_1 = 63.501 \frac{kJ}{mol}$$

MIXING POINT

Steam/Syngas mixture

$$\dot{n}_3, \frac{moles}{hr}$$

T = 260 °C
P = 27 bar
$H_3 = 41.698$

$$\hat{H}_3 = 41.698 \frac{kJ}{mol}$$

$$\dot{n}_2, \frac{mol\ Syngas}{hr}$$

T = 260 °C
P = 27 bar

$$\hat{H}_2 = 18.344 \frac{kJ}{mol}$$

Determine the unknown flow rates for the mixer operating at adiabatic conditions.

Strategy

In order to solve this problem, material balances must be done around the mixing point. Since the compositions of each of the streams are unknown, we also need to perform an energy balance on the system.

Solution

We can start by writing an overall material balance on the system:

$$Input = Output$$
$$\dot{n}_1 + \dot{n}_2 = \dot{n}_3$$

As it can be seen in this equation, there are two unknown variables. Hence, we will need another equation in order to be able to solve this problem.

There is no information given in the problem statement about the composition of the stream but the enthalpies given in the process diagram can be used to perform an energy balance on the system:

$$Q - W_s = \Delta \dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p$$

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The following assumptions can be done for the process described in the problem statement:

$Q = 0$ (adiabatic system)

$W_s = 0$ (the streams are just being added but there are no mechanical parts moving)

$\Delta \dot{E}_p = 0$ (there are no significant height differences)

$\Delta \dot{E}_k = 0$ (the velocity gradients between the input and output streams is negligible)

Thus, the energy equation will be given by:

$$\Delta \dot{H} = 0$$

The change in the enthalpy for this system will be given by:

$$\Delta \dot{H} = \sum_{outlet} \dot{n}_i \hat{H}_i - \sum_{inlet} \dot{n}_i \hat{H}_i = 0$$

Writing this equation in terms of the variables in this problem yields:

$$\Delta \dot{H} = \dot{n}_3 \hat{H}_3 - \left(\underline{\hspace{1.5cm}} + \dot{n}_2 \hat{H}_2 \right) = 0$$

Now we have both the energy and material balance equations, which can be solved simultaneously to determine the flow rates $\dot{n}_2$ and $\dot{n}_3$. Solving for $\dot{n}_2$ from the overall material balance, we get:

$$\dot{n}_2 = \dot{n}_3 - \dot{n}_1$$

Substituting this equation into the energy balance equation gives:

$$\dot{n}_3 \hat{H}_3 - \left[\underline{\hspace{1.5cm}} + \underline{\hspace{1.5cm}} \hat{H}_2 \right] = 0$$

We can enter the quantities for all known variables in this equation to get:

$$\dot{n}_3 \left(\underline{\hspace{1cm}} \frac{kJ}{mol} \right) - \left[\left(8.3 \times 10^6 \frac{mol}{hr} \right) \left(\underline{\hspace{1cm}} \frac{kJ}{mol} \right) + \left(\dot{n}_3 - \underline{\hspace{1cm}} \frac{mol}{hr} \right) \left(\underline{\hspace{1cm}} \frac{kJ}{mol} \right) \right] = 0$$

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This equation can be solved for the flow rate $\dot{n}_3$ as follows:

$$\underline{\hspace{1.5cm}}\,\dot{n}_3 - \underline{\hspace{1.5cm}}\,\dot{n}_3 = \left(8.3\times10^6\,\frac{mol}{hr}\right)\left[\left(63.501\,\frac{kJ}{mol}\right) - \left(18.344\,\frac{kJ}{mol}\right)\right]$$

$$\dot{n}_3 = \frac{\left(8.3\times10^6\,\dfrac{mol}{hr}\right)\left[\left(\underline{\hspace{1.2cm}}\,\dfrac{kJ}{mol}\right)\right]}{\underline{\hspace{1.2cm}}\,\dfrac{kJ}{mol}}$$

$$\boxed{\dot{n}_3 = \underline{\hspace{1.5cm}}\,\frac{mol}{hr}}$$

Now the value of the flow rate $\dot{n}_3$ can be substituted into the material balance equation to yield:

$$\dot{n}_2 = \underline{\hspace{1.5cm}}\,\frac{mol}{hr} - 8.3\times10^6\,\frac{mol}{hr}$$

$$\boxed{\dot{n}_2 = \underline{\hspace{1.5cm}}\,\frac{mol}{hr}}$$