Chapter 2
Introduction to Engineering Calculations

Name: _____________
Date: ______________

The goal of Chapter 2 is to introduce the student to the basics needed to perform engineering calculations, including units, unit conversions, conversion between force and weight, numerical accuracy and precision, dimensional homogeneity, and data analysis. The following problems build upon the fundamentals covered in your text as applied to hydrogen processing, hydrogen as an energy carrier, and the use of hydrogen in fuel cells.

2.2-1 Conversion of Units
2.3-1 Conversion Between Systems of Units
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Example 2.2-1 Conversion of Units

In one year, in the United States there were approximately 4,200,000,000,000 km traveled by 230,000,000 vehicles.

(a) Determine the average number of miles traveled per vehicle.

Strategy

First, we need to determine the number of miles driven. Then we can divide by the number of vehicles to determine an average distance per vehicle.

Solution

Using the "length" table in the front of the textbook we find that:

$$\frac{4.2\times10^{12}\text{ km}}{}\frac{0.6214\text{ mi}}{1\text{km}} = \underline{\hspace{3cm}}\text{ mi}$$

Then, we can divide the number of miles by the number of vehicles to determine the average miles per vehicle.

$$\frac{\underline{\hspace{3cm}}\text{ mi}}{}\frac{}{2.3\times10^{8}\text{ vehicles}} = \underline{\hspace{3cm}}\ \frac{\text{mi}}{\text{vehicle}}$$

(b) If the average fuel economy is 20 miles per gallon, determine the number of gallons needed for the United States for one year.

Strategy

To solve this problem we can divide the number of miles driven by the fuel economy.

Solution

$$\frac{\underline{\hspace{3cm}}\text{ mi}}{}\frac{}{20\dfrac{\text{mi}}{\text{gal}}} = \underline{\hspace{3cm}}\text{ gal}$$

(c) If the average household has 2 vehicles, and if the gasoline cost is \$4 per gallon what is the total cost per household per year? Assume each vehicle travels the national average.

Strategy

This problem can be solved by first determining the total number of gallons per year for the average household. An assumption on the number of miles traveled for each vehicle is required. It is simplest to assume that each car travels the average number of miles. Once this information is provided, the fuel cost can be used to determine household fuel expenses.

Solution

$$\frac{2\,\text{vehicles}}{}\;\frac{\underline{\hspace{3cm}}\,\text{mi}}{\text{vehicle}}\;\frac{}{20\frac{\text{mi}}{\text{gal}}}\;\frac{\$4}{\text{gal}} = \underline{\hspace{3.5cm}}\,\$(\text{fuel cost})$$

(d) One gallon of gasoline has the same amount of energy as a kg of hydrogen. However, fuel cell vehicles have a higher efficiency and can travel 96 km per kilogram of hydrogen. With this information, determine how many billion kg of hydrogen would be needed if all vehicles are replaced by fuel cell vehicles. Also, assume that 40% of the hydrogen is centrally generated at a rate of 1 million kg hydrogen per day and that the rest of the hydrogen is generated at local facilities at a rate of 5000 kg hydrogen per day. How many of each type of production facility are required?

Strategy

This problem can be solved by first determining the total number of kg of hydrogen required and converting to billions of kilograms. Then the required daily production rates from central and local sites can be determined. The throughputs can be used to determine the number of facilities needed.

Solution

$$\frac{4.2\times10^{12}\,\text{km}}{}\;\frac{1\,\text{kg}\,H_2}{96\,\text{km}}\;\frac{\text{billion}\,\text{kg}\,H_2}{10^9\,\text{kg}\,H_2} = \underline{\hspace{3.5cm}}\,\text{billion}\,\text{kg}\,H_2$$

Now knowing the annual hydrogen requirement, the daily production rate can be determined.

$$\frac{\underline{\hspace{3cm}}\,\text{kg}\,H_2}{\text{year}}\;\frac{\text{year}}{365\,\text{days}} = \underline{\hspace{3.5cm}}\,\frac{\text{kg}\,H_2}{\text{day}}$$

The daily rate can be subdivided among large scale and small scale facilities, and knowing the plant production rates the number of facilities can be determined. First, for the large scale facility,

$$\frac{\underline{\hspace{3cm}}\,\text{kg H}_2}{\text{day}}\;\frac{0.40\,\text{kg H}_2\,\text{(large)}}{\text{kg H}_2}\;\frac{\text{large plant}}{\underline{\hspace{3cm}}\,\text{kg H}_2\,\text{(large)/day}} =$$

$$= \underline{\hspace{3cm}}\ \text{large plants}$$

And for the small scale plants we have:

$$\frac{\underline{\hspace{3cm}}\,\text{kg H}_2}{\text{day}}\;\frac{0.60\,\text{kg H}_2\,\text{(small)}}{\text{kg H}_2}\;\frac{\text{small plant}}{\underline{\hspace{3cm}}\,\text{kg H}_2\,\text{(small)/day}} =$$

$$= \underline{\hspace{3cm}}\ \text{small plants}$$

Example 2.3-1 Conversion Between Systems of Units

(a) A hydrogen fuel cell has a volumetric flow rate of 1.1 L/s. Determine the flow rate in ft^3/min.

Strategy

We need to use unit conversions to solve this problem.

Solution

Using the "volume" table in the front of the textbook we find that:

$$\frac{1.1\,\mathrm{L}}{\mathrm{min}}\frac{1\,\mathrm{ft}^3}{28.317\,\mathrm{L}}\frac{\mathrm{min}}{60\,\mathrm{s}} = \underline{\hspace{3cm}}\frac{\mathrm{ft}^3}{\mathrm{s}}$$

(b) The average U.S. household consumes 800 kW-hr of energy each month. Determine the amount of energy in Btu per day.

Strategy

We need to use unit conversions to solve this problem.

Solution

Using the "energy" table in the front of the textbook we find that:

$$\frac{800\,\mathrm{kW\text{-}hr}}{\mathrm{month}}\frac{9.486\times10^{-4}\,\mathrm{Btu}}{2.778\times10^{-7}\,\mathrm{kW\text{-}hr}}\frac{\mathrm{month}}{30\,\mathrm{days}} = \underline{\hspace{3cm}}\frac{\mathrm{Btu}}{\mathrm{day}}$$

Example 2.4-1 Weight and Mass

You have a fuel cell that can produce a maximum of 3.5 kW of electrical power. The device has a mass of 115 lb_m.

(a) Determine the weight of the fuel cell in English and SI units.

Strategy

This problem requires the use of unit conversions and the factor g_c to determine weight in lb_f and N.

Solution

Using the equation $W = mg$ and that g = 9.8066 m/s^2 = 32.174 ft/s^2.

In English units, we have: $W = \dfrac{115\,\mathrm{lb_m}}{} \dfrac{32.174\text{ ft}}{\mathrm{s}^2} \dfrac{1\,\mathrm{lb_f}}{\underline{\hspace{2cm}}\ \mathrm{lb_m}\text{ - ft/s}^2} = \underline{\hspace{3cm}}\ \mathrm{lb_f}$

In SI units, we have:

$$W = \frac{115\,\mathrm{lb_m}}{} \frac{1\text{ kg}}{\underline{\hspace{2cm}}\ \mathrm{lb_m}} \frac{\underline{\hspace{2cm}}\ \mathrm{m}}{\mathrm{s}^2} \frac{1\text{ N}}{\text{kg - m/s}^2} = \underline{\hspace{3cm}}\ \mathrm{N}$$

(b) Determine the "specific power" of the fuel cell in units of W/kg and compare with an internal combustion engine with a specific power of 550 W/kg.

Strategy

This problem requires the use of unit conversions.

Solution

Start with the required ratio and convert units as needed.

$$\frac{3.5\,\mathrm{kW}}{115\,\mathrm{lb_m}} \frac{\underline{\hspace{2cm}}\ \mathrm{W}}{\mathrm{kW}} \frac{\underline{\hspace{2cm}}\ \mathrm{lb_m}}{\mathrm{kg}} = \underline{\hspace{2cm}}\ \frac{\mathrm{W}}{\mathrm{kg}}$$

This value is about ______x less than in the internal combustion engine.

Example 2.5-1 Order-of-Magnitude Estimation

A hydrogen plant produces 1,256,821 kilograms of hydrogen per day.

(a) If the product from this plant is distributed by pipeline to 570 filling stations, use order of magnitude techniques to estimate without a calculator how many cars can fill up at each station if the average demand is 3.13 kilograms per vehicle.

Strategy

To solve this problem we need to write out the proper calculation and then estimate the solution.

Solution

The calculation we need is:

$$\frac{1{,}256{,}821\,\text{kg}\,H_2}{\text{plant - day}}\frac{1\,\text{plant}}{\underline{\qquad}\ \text{stations}}\frac{\text{car}}{3.13\,\text{kg}\,H_2}=$$

Rounding off numbers (and dropping the units), the above is equivalent to:

$$\frac{1.3\times10^6}{\underline{\qquad}\times3}=\frac{1.3\times10^6}{\underline{\qquad}\times10^3}0.7\times10^3\sim\underline{\qquad}$$

This compares with the actual answer of 705 cars per station.

(b) If 83 identical plants run for an average of 275 days per year, estimate the number of billion kg hydrogen produced per year.

Strategy

To solve this problem we need to write out the proper calculation and then estimate the solution.

Solution

The calculation we need is:

$$\frac{1{,}256{,}821\,\text{kg}\,H_2}{\text{plant - day}}\frac{\underline{\qquad}\ \text{plants}}{}\frac{275\,\text{days}}{\text{plant}}\frac{\text{billion kg}\,H_2}{10^9\,\text{kg}\,H_2}=$$

Rounding off numbers (and dropping the units), the above is equivalent to:

$$\frac{1.3\times10^{6}}{}\,\frac{\underline{\quad}\times10^{1}}{}\,\frac{3\times10^{2}}{}\,\frac{}{10^{9}} = 1.3\times\underline{\qquad}\times3 = \underline{\qquad}$$

This compares with the actual answer of 29 billion kg hydrogen.

Example 2.5-2 Statistical Quality Control

(a) The following data is recorded for the voltage of a fuel cell operating at steady state at different points in time.

Voltages (V)
40.69
40.69
40.73
40.73
40.69
40.73
40.59
40.64

Calculate the mean, variance, standard deviation, and the maximum allowed values using 3 standard deviations.

Strategy

To solve this problem we can use the statistical formulas.

Solution

Mean

$$\bar{X} = \frac{1}{N}(X_1 + X_2 + X_3 + ... + X_N)$$
$$\bar{X} = \frac{1}{8}(40.69 + 40.69 + 40.73 + 40.73 + 40.69 + 40.73 + 40.59 + 40.64) = 40.69 \text{ V}$$

Variance

$$\sigma^2 = \frac{1}{N-1}[(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + (X_3 - \bar{X})^2 ... + (X_N - \bar{X})^2]$$
$$\sigma^2 = \frac{1}{8-1}[(40.69 - 40.69)^2 + (40.69 - 40.69)^2 + (40.73 - 40.69)^2 + (40.73 - 40.69)^2$$
$$+ (40.69 - 40.69)^2 + (40.73 - 40.69)^2 + (40.59 - 40.69)^2 + (40.64 - 40.69)^2] = \underline{\qquad} \text{ V}^2$$

Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$
$$\sigma = \sqrt{0.0025} = \underline{\qquad} \text{ V}$$

Maximum Values

______ V x 3 = ± ______ V
40.69 V – ______ V = ______ V
40.69 V + ______ V = ______ V
So the maximum range of values within 3 standard deviations is ______ V – ______ V.

(b) The following data is recorded for the current of a fuel cell operating at steady state at different points in time.

Current (A)
1.47
1.41
1.44
1.49
1.47
1.40
1.41
1.47

Calculate the mean, variance, standard deviation, and the maximum allowed values using 3 standard deviations.

Strategy

To solve this problem we can use the statistical formulas.

Solution

Mean

$$\bar{X} = \frac{1}{N}(X_1 + X_2 + X_3 + ... + X_N)$$
$$\bar{X} = \frac{1}{8}(1.47 + 1.41 + 1.44 + 1.49 + 1.47 + 1.40 + 1.41 + 1.47) = 1.45 \text{ A}$$

Variance

$$\sigma^2 = \frac{1}{N-1}[(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + (X_3 - \bar{X})^2 ... + (X_N - \bar{X})^2]$$
$$\sigma^2 = \frac{1}{8-1}[(1.47-1.45)^2 + (1.41-1.45)^2 + (1.44-1.45)^2 + (1.49-1.45)^2$$
$$+ (1.47-1.45)^2 + (1.40-1.45)^2 + (1.41-1.45)^2 + (1.47-1.45)^2] = \_\_\_\_\_\_ \text{ A}^2$$

Standard Deviation

$\sigma = \sqrt{\sigma^2}$

$\sigma = \sqrt{0.0012} =$ ______ A

Maximum Values

______ A x 3 = ± ______ A
1.45 A – ______ A = ______ A
1.45 A + ______ A = ______ A
So the maximum range of values within 3 standard deviations is ______ A – ______ A.

Example 2.6-1 Dimensional Homogeneity

(a) Consider the Tafel equation for fuel cell voltage V_c as a function of current density i:

$$V_c(\mathrm{V}) = V_{OCV} - A\ln[i(\mathrm{mA/cm^2})/i_o]$$

If the equation is valid, what are the units of the constants V_{OCV}, A, and i_o?

Strategy

We can compare the prescribed units to determine the unknown units.

Solution

The term in the natural log must be dimensionless. Thus, the units on i_o are mA/cm^2. The other terms have units of V_c. Thus, the units on V_{OCV} and A are ______.

(b) Consider the equation for hydrogen consumption m_{H2} in a fuel cell:

$$H_2\, usage(\mathrm{kg/s}) = 1.05\times10^{-8}\frac{P_e(\mathrm{W})}{V_c(\mathrm{V})}$$

If the equation is valid, what are the units of the constant 1.05×10^{-8}?

Strategy

We can compare the prescribed units to determine the unknown units.

Solution

From physics, we know that power is equal to voltage and current. Thus, power in Watts divided by Voltage in volts should have units of current in Amperes. An Ampere (A) is equivalent to a Coulomb (C) per second. Thus, the units on the constant 1.05×10^{-8} has units of kg/C or kg/A-s.

(c) Consider the equation for hydrogen consumption m_{H2} in a fuel cell:

$$H_2\, usage(\mathrm{kg/s}) = 1.05\times10^{-8}\frac{P_e(\mathrm{W})}{V_c(\mathrm{V})}$$

Convert this equation to one where hydrogen usage is reported in mol/hr.

Strategy

We can use unit conversions to determine a new equation for the hydrogen consumption rate.

Solution

Start with the equation and the given units, writing the usage rate as a ratio of mass per unit time.

$$H_2\,usage\,(\text{kg/s}) = \frac{Mass\,(\text{kg})}{Time\,(\text{s})}$$

Then we have:

$$Mass\,(\text{kg}) = Moles\,(\text{mol})\frac{2.02\,(\text{g})}{(\text{mol})}\frac{(\text{kg})}{1000(\text{g})} = \underline{\hspace{3em}}\,Moles$$

And we also have:

$$Time\,(\text{s}) = Minutes(\text{min})\frac{60\,(\text{s})}{(\text{min})} = 60 Minutes$$

Substituting into the left hand side of the formula for hydrogen usage, we have:

$$H_2\,usage\,(\text{kg/s}) = \frac{Mass\,(\text{kg})}{Time\,(\text{s})} = \frac{\underline{\hspace{3em}}\,Moles}{60 Minutes} = \frac{Moles}{\underline{\hspace{3em}}\,Minutes}$$

Inserting into the full formula for hydrogen usage, we have:

$$H_2\,usage\,(\text{mol/min}) = \underline{\hspace{5em}}\,\frac{P_e\,(\text{W})}{V_c\,(\text{V})}$$

Example 2.6-2 Dimensional Homogeneity and Dimensionless Groups

(a) The Nernst equation is used to predict the voltage in a fuel cell as a function of the partial pressure of the gases. This formula can be rearranged to give:

$$\frac{P_{H2}P_{O2}^{1/2}}{P_{H2O}P_o^{1/2}} = \ln\left[\frac{2F(E-E_o)}{RT}\right]$$

If F has units of C/mol e$^-$, E and E_0 have units of V, R has units of J/mol H_2-K, and T has units of K, then what are the units on the constant 2?

Strategy

We can compare the prescribed units to determine the unknown units.

Solution

The term in the natural log must be dimensionless. Thus, noting that 1 V = 1 J/C, we can solve for the units on 2 as:

$$\frac{RT}{F(E-E_o)} = \frac{\dfrac{\text{J}}{\text{mol}\,\text{H}_2-\text{K}}\text{K}}{\dfrac{\text{C}}{\text{mol}\,\text{e}^-}\dfrac{\text{J}}{\text{C}}} = \frac{\text{mol}\,\_\_\_\_}{\text{mol}\,\_\_\_\_}$$

(b) A dimensionless group in heat transfer is the Prandtl number, given by:

$$\text{Pr} = \frac{\mu C_p}{k}$$

If C_p has units of J/g-K and k has units of W/m-K, then what are the units on the viscosity μ?

Strategy

We can compare the prescribed units to determine the unknown units.

Solution

Noting that 1 W = 1 J/s, we can solve for the units on μ as:

$$\mu = \frac{k}{C_p} = \frac{\text{W}}{\text{m}-\text{K}}\frac{\text{g}-\text{K}}{\text{J}}\frac{\text{J/s}}{\text{W}} = \frac{\_\_\_\_\_}{\_\_\_\_\_}$$

Example 2.7-1 Fitting a Straight Line to Flowmeter Calibration Data

(a) A flow rotameter is being used to calibrate a hydrogen feed to a fuel cell. The data are given below.

Flow Rate (SLPM)	Rotameter Reading
0.21	10
0.61	25
0.89	35
1.17	50

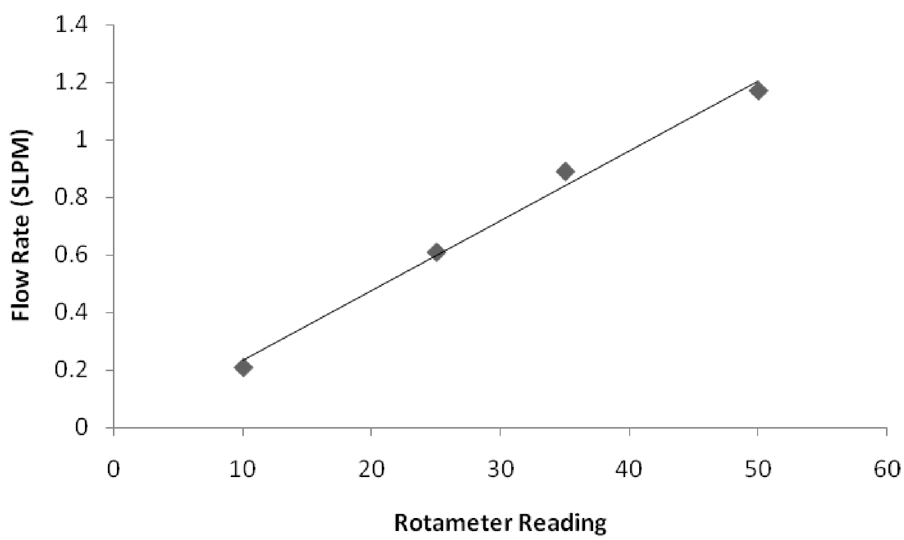
Determine a linear equation to predict the hydrogen flow rate as a function of rotameter reading and estimate the flow rate for a rotameter reading of 42.

Strategy

We can plot a line and determine its slope and intercept to give a linear fit.

Solution

The plot of the data is illustrated below with a best-fit line.



We can extrapolate the line to give the intercept as approximately 0. The slope of the line can be determined from the rise over the run of the data according to:

$$\text{slope} = \frac{(1.17-0.21)}{(50-10)} = \frac{\_\_\_\_\_\_}{40} = \_\_\_\_\_\_$$

Thus, we have Flow Rate = ______ x Rotameter Reading. For a reading of 42, the Flow Rate is estimated as ______ x 42 = ______ SLPM.

(b) A flow rotameter is being used to calibrate an air feed to a fuel cell. The data are given below:

Flow Rate (SLPM)	Rotameter Reading
1.5	18
2.3	28
3.1	40
4.2	55

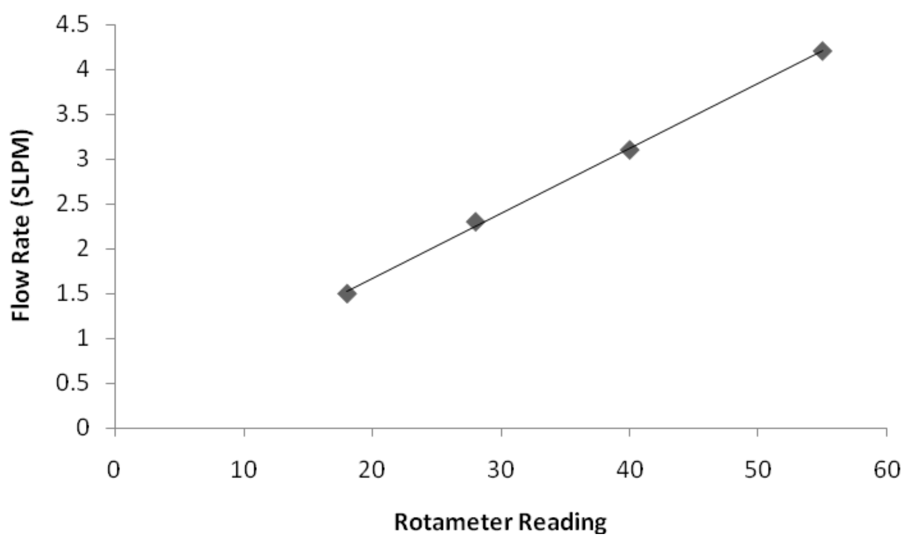
Determine a linear equation to predict the air flow rate as a function of rotameter reading and estimate the flow rate for a rotameter reading of 33.

Strategy

We can plot a line and determine its slope and intercept to give a linear fit.

Solution

The plot of the data is illustrated below with a best-fit line.



We can extrapolate the line to give the intercept as approximately 0.25. The slope of the line can be determined from the rise over the run of the data according to:

$$\text{slope} = \frac{(4.2-1.5)}{(55-18)} = \frac{\_\_\_\_\_\_}{37} = \_\_\_\_\_\_$$

Thus, we have Flow Rate = 0.25 + ______ x Rotameter Reading. For a reading of 33, the Flow Rate is estimated as 0.25 + ______ x 33 = ______ SLPM.

Example 2.7-2 Linear Curve-Fitting of Nonlinear Data

(a) The diffusion coefficient of oxygen in nitrogen is needed for design of fuel cell diffusion layers. The diffusion coefficient in cm^2/s is shown as a function of temperature in degrees Kelvin in the table below.

Temperature (K)	**Diffusion Coefficient (cm^2/s)**	**$T^{3/2}$ (for model)**
298	0.20	5140
328	0.22	5940
358	0.27	6770
398	0.31	7940

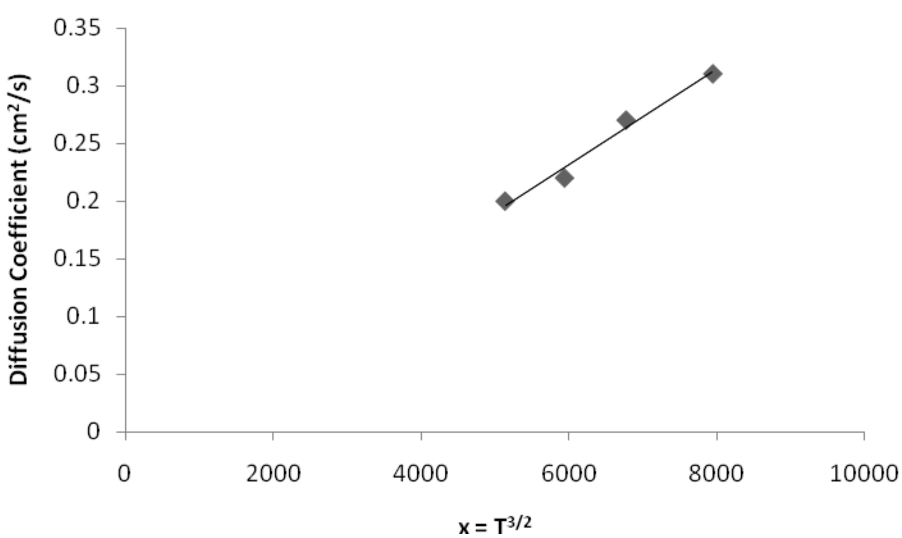
It has been suggested that the data can be modeled by $D = AT^{3/2} + B$. Determine the values of *A* and *B*.

Strategy

We can convert the equation to the form of a straight line, then develop a plot to determine the slope and intercept. From this data, *A* and *B* can be determined.

Solution

If we define $y = D$ and $x = T^{3/2}$, we have $y = Ax + B$. An x-y plot is shown below:



At a glance, it appears the data goes nearly through the origin, such that $B = 0$. The slope of the data is estimated from the first and last data points as:

$$\text{slope} = \frac{(0.31-0.20)}{(7940-5140)} = \frac{\_\_\_\_\_\_\_\_}{2800} = \_\_\_\_\_\_\_\_\_\_\_\_ \frac{\text{cm}^2/\text{s}}{\text{K}^{3/2}}$$

Thus, the model is given by: $D = \_\_\_\_\_\_\_\_\_\_ T^{3/2}$.

(b) Data has been collected for the power to operate an air compressor in a fuel cell, and is shown in the table below as a function of the compression ratio.

Compression Ratio	Compressor Power (kW)	Compression Ratio$^{0.29}$ (for model)
2.0	80	1.2
3.0	135	1.4
4.0	190	1.5
5.0	215	1.6

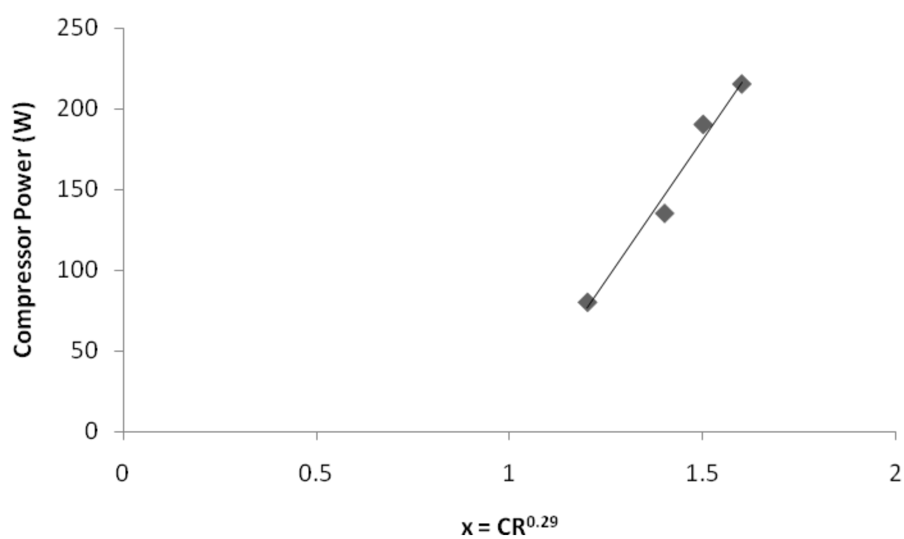
It has been suggested that the data can be modeled by $P = A \times CR^{0.29} + B$. Determine the values of A and B.

Strategy

We can convert the equation to the form of a straight line, then develop a plot to determine the slope and intercept. From this data, A and B can be determined.

Solution

If we define $y = P$ and $x = CR^{0.29}$, we have $y = Ax + B$. An x-y plot is shown below:



A glance at the data shows the data crosses the x-axis near x = 1. Thus, if we substitute into the linear equation we have: $0 = A + B$ such that $B \sim -A$.

We can determine the slope of the data to get *A* according to:

$$\text{slope} = \frac{(215-80)}{(1.6-1.2)} = \frac{\underline{\qquad\quad}}{0.4} = \underline{\qquad\qquad} \text{ kW}$$

Thus, the model is given by: $P = \underline{\qquad\qquad}(CR^{0.29} - 1)$.

Example 2.7-3 Curve Fitting on Semilog and Log Plots

(a) In 2003 the energy consumption in the United States of America was 98.3 Quadrillion Btu (also called quads). In 1998, the amount was 95.2 quads. Determine the values of the constants in a fit of the form:

$Q = A\exp(Bt)$ and $Q = Ct^D$ where Q is energy consumption in quads and t is time in years.

Strategy

With the data we have two equations and two unknowns, so algebra can be used to determine the solution.

Solution

For the case of exponential growth, we have $Q_1 = A\exp(Bt_1)$ and $Q_2 = A\exp(Bt_2)$. Dividing the equations, and taking the natural logarithm of both sides gives a solution for B as:

$$B = \frac{\ln(Q_1 / Q_2)}{t_1 - t_2}$$

With the given data we have:

$$B = \frac{\ln(98.3/95.2)}{2003-1998} = \underline{\hspace{3em}} \text{ yr}^{-1}$$

And we can then solve for A from:

$$A = Q_1 \exp(-Bt_1) = 98.3\exp(-\underline{\hspace{3em}}\times 2003) = \underline{\hspace{3em}} \text{ QBtu}$$

For the case of geometric growth, we have $Q_1 = Ct_1{}^D$ and $Q_2 = Ct_2{}^D$. Dividing the equations, and taking the natural logarithm of both sides gives a solution for D as:

$$D = \frac{\ln(Q_1 / Q_2)}{\ln(t_1 / t_2)}$$

With the given data we have:

$$D = \frac{\ln(98.3/95.2)}{\ln(2003/1998)} = \underline{\hspace{3em}}$$

And we can then solve for C from:

$$C = Q_1 t_1^{-D} = 98.3 \times 2003^{-\_\_\_\_\_} = \_\_\_\_\_\_\_\_ \text{ QBtu yr}^{-\_\_\_\_\_}$$

(b) In 2003 the energy consumption in the United States of America was 98.3 Quadrillion Btu (also called quads). In 1983, the amount was 73.0 quads. Determine the values of the constants in a fit of the form:

$Q = A\exp(Bt)$ and $Q = Ct^D$ where Q is energy consumption in quads and t is time in years.

Strategy

With the data we have two equations and two unknowns, so algebra can be used to determine the solution.

Solution

For the case of exponential growth, we have $Q_1 = A\exp(Bt_1)$ and $Q_2 = A\exp(Bt_2)$. Dividing the equations, and taking the natural logarithm of both sides gives a solution for B as:

$$B = \frac{\ln(Q_1 / Q_2)}{t_1 - t_2}$$

With the given data we have:

$$B = \frac{\ln(98.3/73.0)}{2003 - 1983} = \_\_\_\_\_\_\_\_ \text{ yr}^{-1}$$

And we can then solve for A from:

$$A = Q_1 \exp(-Bt_1) = 98.3\exp(-\_\_\_\_\_\_ \times 2003) = \_\_\_\_\_\_\_\_ \text{ QBtu}$$

For the case of geometric growth, we have $Q_1 = Ct_1^D$ and $Q_2 = Ct_2^D$. Dividing the equations, and taking the natural logarithm of both sides gives a solution for D as:

$$D = \frac{\ln(Q_1 / Q_2)}{\ln(t_1 / t_2)}$$

With the given data we have:

$$D = \frac{\ln(98.3/73.0)}{\ln(2003/1983)} = \underline{\qquad\quad}$$

And we can then solve for C from:

$$C = Q_1 t_1^{-D} = 98.3 \times 2003^{-\underline{\qquad}} = \underline{\qquad\quad} \text{ QBtu yr}^{-\underline{\qquad}}$$