Chapter 11   Name: _____
Balances on Transient Processes   Date: _____

Chapter 11 contains examples where the process conditions and variables change with time. The main difference with the problems explained in previous chapters is that now the accumulation term will have to be considered when performing material balances. The following example problems explain how to derive material and energy balances on transient systems applied to hydrogen technology and fuel cells.

Daniel López Gaxiola   Student View
Jason M. Keith

Example 11.1-1 Differential Balances on a Fuel Cell

One of the critical parameters affecting proton-exchange membrane fuel cells is the humidity of the electrolyte membrane. Before starting a fuel cell vehicle, air with a relative humidity of 0.8 is fed into the fuel cell with the purpose of humidifying the membrane so hydrogen protons can move through it to the cathode side. A diagram of this process is shown in the following figure:

Humid Air

$$\dot{\upsilon}_0\left(\frac{L}{s}\right), \rho_0\left(\frac{g}{L}\right)$$

$$h_r = 0.8$$

Anode		Cathode

Humid Air

$$\dot{\upsilon}\left(\frac{L}{s}\right), \rho\left(\frac{g}{L}\right)$$

$$h_r \leq 0.8$$
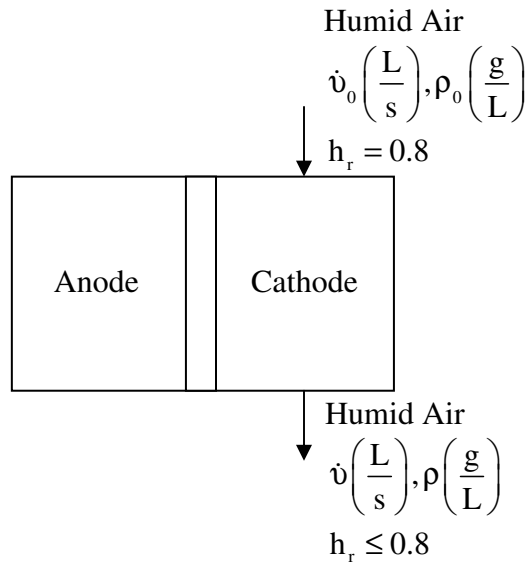
Perform a differential balance on the fuel cell for the membrane humidification process.

Strategy

We will have to make use of the general material balance equation to evaluate the amount of water mass M accumulated inside the fuel cell.

Solution

The general balance equation is given by:

$$\text{Accumulation} = \text{Input} - \text{Output} + \text{Generation} - \text{Consumption}$$

Since there is no chemical reaction occurring in the humidification of the polymer-electrolyte membrane, the generation and consumption terms can be neglected from the material balance equation, to get:

$$\text{Accumulation} = \text{Input} - \text{Output}$$

Rewriting this equation in terms of the properties of the air entering the fuel cell, we have:

Daniel López Gaxiola
Jason M. Keith

Supplemental Material for Elementary Principles of Chemical Processes

$$\text{Accumulation} = \frac{dM}{dt}\left(\frac{g}{s}\right)$$

$$\text{Input} = \rho_0\left(\frac{g}{L}\right)\dot{\upsilon}_0\left(\frac{L}{s}\right)$$

$$\text{Output} = \rho\left(\frac{g}{L}\right)\dot{\upsilon}\left(\frac{L}{s}\right)$$

The accumulation term can be rewritten as follows:

$$\text{Accumulation} = \frac{dM}{dt} = \frac{d(\rho V)}{dt}$$

Applying the chain rule, the accumulation term will be given by:

$$\text{Accumulation} = \frac{d(\rho V)}{dt} = \underline{\quad}\frac{dV}{dt} + V\overline{\overline{\frac{}{dt}}}$$

Substituting the corresponding equations into the general balance equation yields:

$$\underline{\quad}\frac{dV}{dt} + V\overline{\overline{\frac{}{dt}}} = \rho_0\dot{\upsilon}_0 - \underline{\quad\quad}$$

The volume V is the volume of cathode chamber and membrane in the fuel cell. Since this is a constant value, the change in this volume with respect to time is equal to zero. Thus,

$$\boxed{V\overline{\overline{\frac{}{dt}}} = \underline{\quad\quad}}$$

According to this equation, the density is changing with respect to time. This is due to the water being retained in the fuel cell by the membrane.

Daniel López Gaxiola
Jason M. Keith

Supplemental Material for Elementary Principles of Chemical Processes

Example 11.1-2 Hydrogen Balance on a Fuel Tank

A vehicle operating with a stack of proton-exchange membrane fuel cells is consuming hydrogen according to the driving schedule shown below. The hydrogen consumption rate as a function of time was obtained based on the fuel consumed by a family vehicle during the winter school period, including warm-up times and assuming the vehicle is running at an average speed of $25\dfrac{\text{mi}}{\text{hr}}$ and has

a fuel economy of $50\dfrac{\text{mi}}{\text{kg H}_2}$.

The consumption rate vs. time is shown in the following figure, where the line is a best fit of the hydrogen consumption rate given by $0.5477t^{-0.175}\dfrac{\text{kg}}{\text{hr}}$, where t is the vehicle operation time in hours.
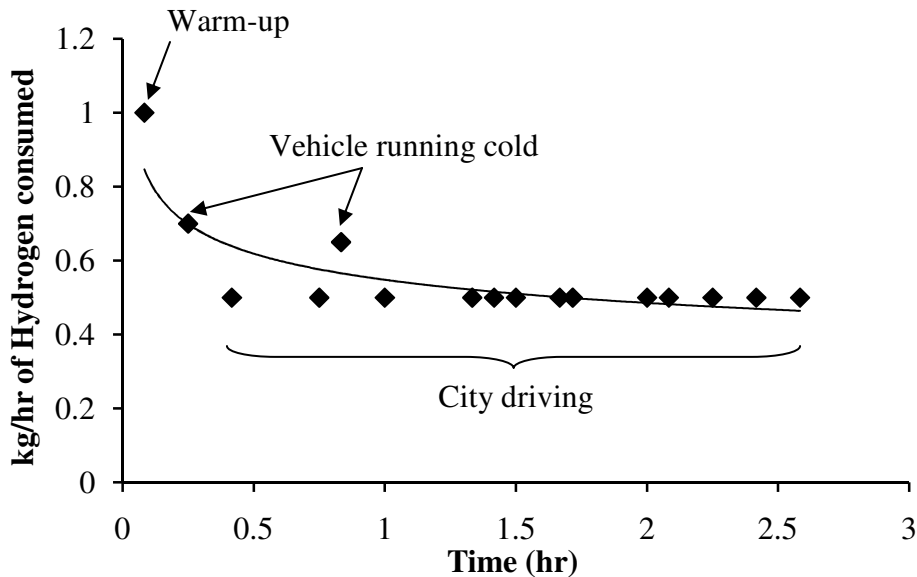
By using differential balances on the tank, determine the amount of fuel remaining in the tank after running the vehicle for 6 hours, assuming that the fuel tank is initially at its maximum capacity of 5 kg of hydrogen.

Strategy

Material balances can be performed on the hydrogen tank by using the fuel consumption as a function of time.

Solution

We can start by writing the material balance equation for a general system:

Daniel López Gaxiola   Student View
Jason M. Keith

Supplemental Material for Elementary Principles of Chemical Processes

$$\frac{dM}{dt} = \dot{m}_{in} - \dot{m}_{out} + \dot{r}_{gen} - \dot{r}_{cons}$$

where:

$\dfrac{dM}{dt}$ = change in hydrogen mass inside the tank with respect to time

$\dot{m}_{in}$ = hydrogen mass flow rate entering the tank

$\dot{m}_{out}$ = hydrogen mass flow rate exiting the tank

$\dot{r}_{gen}$ = mass flow rate of H_2 generated by the reaction.

$\dot{r}_{cons}$ = mass flow rate of H_2 consumed by the reaction.

Since the tank is initially full and there is no chemical reaction taking place inside the tank, the material balance equation can be reduced to:

$$\frac{dM}{dt} = -\dot{m}_{out}$$

Substituting the mass flow rate of hydrogen as a function of time, we get:

$$\frac{dM}{dt} = \underline{\hspace{2cm}} \frac{kg}{hr}$$

This differential equation can be separated to yield:

$$dM = \underline{\hspace{2.5cm}} dt$$

It is known that initially, the tank is at its full capacity. This will be the initial condition used to solve the integral, as shown in the following steps:

$$\int_{\underline{\hspace{0.5cm}}kg}^{M} dM = \int_{0}^{t} \underline{\hspace{2cm}} dt$$

$$M\Big|_{\underline{\hspace{0.5cm}}kg}^{M} = \underline{\hspace{2cm}}\Big|_{0}^{t}$$

After applying the initial condition to the integrated mass balance equation, we have:

$$M - 5 = \underline{\hspace{2cm}} - \left[-0.6639(0)^{0.825} \right]$$

$$M = \underline{\hspace{2cm}} + 5$$
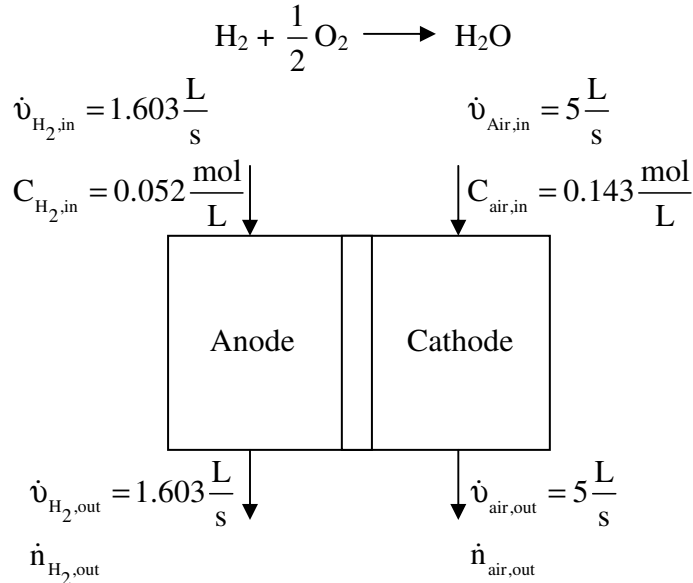
Daniel López Gaxiola
Jason M. Keith

This equation allows us to calculate the amount of fuel remaining in the tank at a certain time. If the vehicle runs for 6 hours, the remaining fuel will be given by:

$$M = \underline{\hspace{3cm}} + 5$$

$$\boxed{M = \underline{\hspace{1.5cm}} \ kg \ H_2}$$

Supplemental Material for Elementary Principles of Chemical Processes

Example 11.2-2 Transient Behavior of a Proton-Exchange Membrane Fuel Cell

The following diagram is describing the reaction and process occurring inside a one-cell proton-exchange membrane fuel cell:

$$H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$$

$$\dot{v}_{H_2,in} = 1.603 \frac{L}{s} \qquad\qquad \dot{v}_{Air,in} = 5\frac{L}{s}$$

$$C_{H_2,in} = 0.052\frac{mol}{L} \qquad\qquad C_{air,in} = 0.143\frac{mol}{L}$$

Anode		Cathode

$$\dot{v}_{H_2,out} = 1.603\frac{L}{s} \qquad\qquad \dot{v}_{air,out} = 5\frac{L}{s}$$

$$\dot{n}_{H_2,out} \qquad\qquad\qquad \dot{n}_{air,out}$$

The current produced by the fuel cell is a function of the amount of hydrogen reacted, and can be determined by the following equation:

$$\dot{n}_{H_2,reacted} = \frac{IN}{2F}$$

where:

I = Current produced by the fuel cell in amperes (A)
N = Number of fuel cells (here we have assumed 1 cell)
F = Faraday's constant = $96485\dfrac{C}{mol \cdot e^-}$

Derive an equation for the current produced by the fuel cell as a function of time if 93 % of the hydrogen is converted into products. Assume that the chemical reaction occurring in the fuel cell is a first order reaction.

Strategy

We can use material balances for hydrogen on the reactor and solve for the current I.

Daniel López Gaxiola
Jason M. Keith

Student View

Supplemental Material for Elementary Principles of Chemical Processes

Solution

The general balance equation is given by:

$$\text{Accumulation} = \text{Input} - \text{Output} + \text{Generation} - \text{Consumption}$$

In this problem, each one of the terms in the material balance equation is described by the following equations:

$$\text{Accumulation} = \frac{dn_{H_2}}{dt}$$

$$\text{Input} = \dot{n}_{H_2,in} = \dot{\upsilon}_{H_2,in} C_{H_2,in}$$

$$\text{Output} = \dot{n}_{H_2,out} = \dot{n}_{H_2,in} - \dot{n}_{H_2,reacted}$$

$$\text{Generation} = 0$$

$$\text{Consumption} = \dot{n}_{H_2,reacted} = \frac{IN}{2F}$$

Substituting each equation in the material balance equation, we get:

$$\frac{dn_{H_2}}{dt} = \dot{n}_{H_2,in} - \dot{n}_{H_2,out} + 0 - \dot{n}_{H_2,reacted}$$

It is known that 93 % of the hydrogen entering the reactor is converted into hydrogen. Thus,

$$\dot{n}_{H_2,out} = \dot{n}_{H_2,in} - \underline{\qquad} = 0.07\dot{n}_{H_2,in}$$

This equation can be entered into the material balance equation to yield:

$$\frac{dn_{H_2}}{dt} = \underline{\qquad} - 0.07\dot{n}_{H_2,in} - \dot{n}_{H_2,reacted}$$

Writing the amount of hydrogen reacted in terms of the current, gives:

$$\frac{dn_{H_2}}{dt} = \underline{\qquad} - \frac{IN}{2F}$$

Daniel López Gaxiola
Jason M. Keith

Student View

Supplemental Material for Elementary Principles of Chemical Processes

The reaction rate indicates the consumption rate of hydrogen. For a first order reaction, the rate will be given by:

$$r = kn_{H_2}$$

where:

k = reaction rate constant in s^{-1}

The amount of hydrogen reacted can be obtained also by using the rate equation:

$$kn_{H_2} = \dot{n}_{H_2,reacted}$$

Writing the consumption rate of hydrogen in terms of the current, we get:

$$kn_{H_2} = \underline{\hspace{2cm}}$$

This equation can be differentiated to obtain the change of moles of hydrogen with time, as a function of the change in the current with respect to time:

$$k\frac{dn_{H_2}}{dt} = \underline{\hspace{1.5cm}} \frac{dI}{dt}$$

Solving for $\dfrac{dn_{H_2}}{dt}$, we have:

$$\frac{dn_{H_2}}{dt} = \underline{\hspace{1.5cm}} \frac{dI}{dt}$$

Substituting this equation into the mass balance equation yields:

$$\underline{\hspace{1.5cm}} \frac{dI}{dt} = \underline{\hspace{1.5cm}} - \frac{IN}{2F}$$

Solving for the change in the current with time, we have:

$$\frac{dI}{dt} = \underline{\hspace{1.5cm}} - kI$$

Entering all the known quantities into this equation gives the following result:

Daniel López Gaxiola   Student View
Jason M. Keith

Supplemental Material for Elementary Principles of Chemical Processes

$$\frac{dI}{dt} = \frac{2\left(\dfrac{C}{\text{mol}\cdot e^-}\right)k}{1} \cdot 0.93\left[\left(\dfrac{\rule{1cm}{0.4pt}}{s}\ L\right)\left(0.052\dfrac{\text{mol}}{L}\right)\right] - kI$$

$$\frac{dI}{dt} = \underline{\hspace{1.5cm}}\ k - kI$$

Initially, there is no hydrogen reacting in the fuel cell. Thus, the current generated is equal to zero:

@t=0, I = 0

Separating and integrating the equation for the rate of change for the current, we get:

$$\frac{dI}{\underline{\hspace{1cm}}\ k - kI} = dt$$

$$\int_0^I \frac{dI}{\underline{\hspace{1cm}}\ k - kI} = \int_0^t dt$$

$$-\frac{1}{k}\left[\ln\left(\underline{\hspace{1cm}}\ k - kI\right) - \underline{\hspace{1.5cm}}\right] = t$$

Using logarithmic equation properties, this equation can be rewritten as follows:

$$\ln\frac{\left(\underline{\hspace{1cm}}\ k - kI\right)}{\left(14959k\right)} = -kt$$

Taking the exponential of both left and right hand sides of this equation, eliminates the natural logarithm on the left side and allows us to solve for the current I as shown in the following steps.

$$\underline{\hspace{1.5cm}}\ k - kI = 14959ke^{-kt}$$

$$\underline{\hspace{1.5cm}} - I = \frac{14959ke^{-kt}}{\underline{\hspace{0.7cm}}}$$

$$\boxed{I = \underline{\hspace{3cm}}}$$

In a real fuel cell, the reaction rate is more complicated, but this example has shown how there is a delay in the response to load changes in a fuel cell.

Daniel López Gaxiola
Jason M. Keith

Student View

Example 11.3-1 Startup of a Fuel Cell

A fuel cell vehicle tank is being filled at a hydrogen fuel station to its maximum capacity of 5.3 kg. The fuel cell is initially at an ambient temperature of 35°C. At steady state the reactants are at a temperature of 80°C.

The cell is initially loaded with 21 moles of hydrogen and 50 moles of air at 35°C. The total mass of the fuel cell stack is 22 kg and the heat capacity of the bipolar plates in the fuel cell is $1.607 \times 10^{-3} \frac{kJ}{g \cdot °C}$. The bipolar plates are the component of the fuel cell that give structural support to the fuel cell stack and separate the cathode side of a fuel cell from the anode side of the adjacent fuel cell.

Determine the amount of heat that must be transferred to the reactants to reach 80°C if the warm-up time is 5 minutes.

Strategy

The energy balance equation for a closed system can be applied to determine the heat transfer rate to the fuel cell.

Solution

The energy balance equation for a closed system is given by the equation shown below:

$$m_T C_v \frac{dT}{dt} = \dot{Q} - \dot{W}$$

where:

   m_T = Total mass in the system

   C_v = Heat capacity of the system at constant volume

Since the mixing of the gas streams in the fuel cells is achieved due to the channels engraved on the bipolar plates, there are no mechanical parts moving to mix the gases. Thus, the work $\dot{W}$ is neglected from the energy balance equation.

The initial condition for this system is given by:

   $t = 0, T = 35°C$

To solve this problem, the energy balance equation must be integrated from the initial conditions of the fuel cell to the warm-up time of 5 minutes and the reaction temperature of 80°C.

Supplemental Material for Elementary Principles of Chemical Processes

The heat capacity of the system C_V will be assumed constant, since there are no considerable changes in temperature for this process. Thus, the heat capacity of the system is calculated as follows:

$$\tilde{C}_v = x_{H_2}\tilde{C}_{v,H_2} + x_{Air}\tilde{C}_{v,Air} + x_{Fuel\ cell\ stack}\tilde{C}_{v,Bipolar\ plates}$$

where:

x_i = mass fraction of component i

$\tilde{C}_{v,i}$ = mass heat capacity at constant volume of the component i

It can be seen that the heat capacity of the bipolar plates will be used in the calculation of the heat capacity of the system, since it is the component directly exposed to the fuel cell surroundings.

The heat capacity of the air and the hydrogen will be calculated using the parameters in Table B.2 of Felder & Rousseau at the average temperature between the initial and final conditions of the system, obtained as follows:

$$\overline{T} = \frac{T_{initial} + T_{final}}{2} = \frac{35°C + 80°C}{2}$$

$$\overline{T} = \underline{} °C$$

From Table B.2:

$$\hat{C}_{p,air}\left(\frac{kJ}{mol \cdot °C}\right) = (\underline{}) + (4.147 \times 10^{-6})T + (3.191 \times 10^{-9})T^2 - (\underline{})T^3$$

$$\hat{C}_{p,H_2}\left(\frac{kJ}{mol \cdot °C}\right) = (2.884 \times 10^{-2}) + (\underline{})T + (\underline{})T^2 - (8.698 \times 10^{-13})T^3$$

These are the equations for calculating the heat capacity of air and hydrogen at constant pressure. However, we need to calculate the heat capacity at constant volume from these equations. To do this, we need to make use of the following equation:

$$\hat{C}_v = \hat{C}_p - R$$

where:

$$R = universal\ gas\ constant = 8.314 \times 10^{-3}\ \frac{kJ}{mol \cdot °C}$$

Daniel López Gaxiola   Student View
Jason M. Keith

Supplemental Material for Elementary Principles of Chemical Processes

Thus, the heat capacities at constant volume will be given by:

$$\hat{C}_{v,air}\left(\frac{kJ}{mol\cdot°C}\right) = (\underline{\hspace{1.5cm}}) + (4.147\times10^{-6})T + (3.191\times10^{-9})T^2 - (\underline{\hspace{1.5cm}})T^3$$
$$-(8.314\times10^{-3})$$

$$\hat{C}_{v,H_2}\left(\frac{kJ}{mol\cdot°C}\right) = (2.884\times10^{-2}) + (\underline{\hspace{1.5cm}})T + (\underline{\hspace{1.5cm}})T^2 - (8.698\times10^{-13})T^3$$
$$-(8.314\times10^{-3})$$

Substituting the average temperature value into these two equations yields:

$$\hat{C}_{v,air}\left(\frac{kJ}{mol\cdot°C}\right) = (\underline{\hspace{1.5cm}}) + (4.147\times10^{-6})(\underline{\hspace{0.5cm}}°C) + (3.191\times10^{-9})(\underline{\hspace{0.5cm}}°C)^2$$
$$-(\underline{\hspace{1.5cm}})(\underline{\hspace{0.5cm}}°C)^3$$

$$\hat{C}_{v,air} = 2.088\times10^{-2}\frac{kJ}{mol\cdot°C}$$

$$\hat{C}_{v,H_2}\left(\frac{kJ}{mol\cdot°C}\right) = (2.053\times10^{-2}) + (\underline{\hspace{1.5cm}})(\underline{\hspace{0.5cm}}°C)$$
$$+(\underline{\hspace{1.5cm}})(\underline{\hspace{0.5cm}}°C)^2 - (8.698\times10^{-13})(\underline{\hspace{0.5cm}}°C)^3$$

$$\hat{C}_{v,H_2} = \underline{\hspace{1.5cm}}\frac{kJ}{mol\cdot°C}$$

If we look at the equation for the heat capacity of the system, it can be seen that the mass fractions of each gas are multiplying their corresponding heat capacity. Hence, we must convert these molar heat capacity values into mass units by dividing by the molecular weight:

$$\tilde{C}_{v,air} = \frac{\hat{C}_{v,air}}{M_{air}} = \frac{2.088\times10^{-2}\frac{kJ}{mol\cdot°C}}{28.84\frac{g}{mol}} = \underline{\hspace{1.5cm}}\frac{kJ}{g\cdot°C}$$

Supplemental Material for Elementary Principles of Chemical Processes

$$\tilde{C}_{v,H_2} = \frac{\hat{C}_{v,H_2}}{M_{H_2}} = \frac{\dfrac{kJ}{mol \cdot °C}}{2\dfrac{g}{mol}} = \underline{\hspace{1cm}} \frac{kJ}{g \cdot °C}$$

In order to determine the mass fraction of each component, we need to calculate first the total mass of the system:

$$m_T = m_{H_2} + m_{Air} + m_{Fuel\ cell\ stack} = \underline{\hspace{0.8cm}} \text{ moles } H_2 \left(\frac{1 \text{ kmol } H_2}{1000 \text{ moles } H_2} \right) \left(\frac{2 \text{ kg}}{1 \text{ kmol } H_2} \right)$$

$$+ \underline{\hspace{0.8cm}} \text{ moles air} \left(\frac{1 \text{ kmol air}}{1000 \text{ moles air}} \right) \left(\frac{28.84 \text{ kg}}{1 \text{ kmol air}} \right) + \underline{\hspace{0.5cm}} \text{ kg}$$

$$m_T = \underline{\hspace{1.5cm}} \text{ kg}$$

The mass fraction of each gas can be obtained by dividing the mass of each individual substance by the total mass of the system. Thus,

$$x_{H_2} = \frac{m_{H_2}}{m_T} = \frac{0.042 \text{ kg } H_2}{\underline{\hspace{1.5cm}} \text{ kg}} = \underline{\hspace{1.5cm}}$$

$$x_{Air} = \frac{m_{air}}{m_T} = \frac{\underline{\hspace{1.5cm}} \text{ kg air}}{\underline{\hspace{1.5cm}} \text{ kg}} = 0.0614$$

$$x_{Fuel\ cell\ stack} = \frac{m_{Fuel\ cell\ stack}}{m_T} = \frac{\underline{\hspace{1cm}} \text{ kg}}{\underline{\hspace{1.5cm}} \text{ kg}} = \underline{\hspace{1.5cm}}$$

Now all the calculated mass fractions and heat capacity, and the heat capacity of bipolar plates given in the problem statement can be entered into the equation for the heat capacity of the system at constant volume to yield:

$$\tilde{C}_v = x_{H_2} \tilde{C}_{v,H_2} + x_{Air} \tilde{C}_{v,Air} + x_{Fuel\ cell\ stack} \tilde{C}_{v,Bipolar\ plates}$$

$$\tilde{C}_v = \underline{\hspace{1cm}} \left(1.028 \times 10^{-2} \frac{kJ}{g \cdot °C} \right) + 0.0614 \left(\underline{\hspace{1cm}} \frac{kJ}{g \cdot °C} \right) + \underline{\hspace{1cm}} \left(\underline{\hspace{1cm}} \frac{kJ}{g \cdot °C} \right)$$

$$\tilde{C}_v = \underline{\hspace{1.5cm}} \frac{kJ}{g \cdot °C}$$

Supplemental Material for Elementary Principles of Chemical Processes

Substituting this heat capacity value and the total mass of the system in the energy balance equation yields:

$$\left(23.484 \text{ kg}\right)\left(\frac{1000 \text{ g}}{1 \text{ kg}}\right)\left(\underline{\hspace{1cm}} \frac{\text{kJ}}{\text{g} \cdot {}^{\circ}\text{C}}\right)\frac{dT}{dt} = \dot{Q}$$

$$\left(\underline{\hspace{1cm}} \frac{\text{kJ}}{{}^{\circ}\text{C}}\right)\frac{dT}{dt} = \dot{Q}$$

This equation can be separated and integrated from the initial to the final conditions of the fuel cell, as shown in the following steps. The heat transfer rate $\dot{Q}$ is assumed to be constant.

$$\left(\underline{\hspace{1cm}} \frac{\text{kJ}}{{}^{\circ}\text{C}}\right)\int_{T=35^{\circ}\text{C}}^{T=\underline{\hspace{0.5cm}}{}^{\circ}\text{C}} dT = \dot{Q}\int_{t=0 \text{ min}}^{t=5 \text{ min}} dt$$

$$\left(\underline{\hspace{1cm}} \frac{\text{kJ}}{{}^{\circ}\text{C}}\right)\left(T\big|_{35^{\circ}\text{C}}^{\underline{\hspace{0.5cm}}{}^{\circ}\text{C}}\right) = \dot{Q}\left(t\big|_{0 \text{ min}}^{5 \text{ min}}\right)$$

$$\left(\underline{\hspace{1cm}} \frac{\text{kJ}}{{}^{\circ}\text{C}}\right)\left(\underline{\hspace{1cm}}{}^{\circ}\text{C} - \underline{\hspace{1cm}}{}^{\circ}\text{C}\right) = \dot{Q}\left(5 \text{ min}\right)$$

Solving for the heat transfer rate $\dot{Q}$, we have:

$$\dot{Q} = \frac{\left(\underline{\hspace{1cm}} \frac{\text{kJ}}{{}^{\circ}\text{C}}\right)\left(\underline{\hspace{1cm}}{}^{\circ}\text{C}\right)}{\left(5 \text{ min}\right)}\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$\boxed{\dot{Q} = \underline{\hspace{1cm}} \text{ kW}}$$

(b) What percent of the total amount of hydrogen would need to be combusted to heat the reaction gases?

Strategy

We will need to use the heat of combustion to determine how many moles of hydrogen will be consumed after heating the fuel cell to the temperature of 80°C.

Solution

The heat of combustion of hydrogen can be obtained from Table B.1 of Felder & Rousseau to be:

Supplemental Material for Elementary Principles of Chemical Processes

$$\Delta H^o_{C,H_2} = \underline{\qquad} \frac{kJ}{mol}$$

The total amount of heat that can be produced from the hydrogen in the tank can be calculated by multiplying the amount of hydrogen by the heat of combustion:

$$H_{H_2} = 5.3 \ kg \left(\frac{1 \ kmol \ H_2}{2 \ kg} \right) \left(\frac{1000 \ mol}{1 \ kmol} \right) \left(\underline{\qquad} \frac{kJ}{mol} \right)$$

$$H_{H_2} = \underline{\qquad} kJ$$

In part a) we calculated that the required rate of heat transfer to the reactants in the fuel cell is equal to _____ kW. In order to determine the amount of energy from this heat transfer rate, we need to multiply it by the warm-up time:

$$Q = t_{warm-up} \dot{Q}$$

Substituting the warm-up time of 5 minutes and the heat transfer rate in this equation yields:

$$Q = (\underline{\quad} \ min) \left(\frac{60 \ s}{1 \ min} \right) \left(\underline{\qquad} \frac{kJ}{s} \right)$$

$$Q = \underline{\qquad} kJ$$

Finally, we can determine the percent of hydrogen consumed by dividing the required heat by the total heat available:

$$\% \ Hydrogen \ consumed = \frac{Q}{H_{H_2}} \times 100$$

$$\% \ Hydrogen \ consumed = \frac{\underline{\qquad} \ kJ}{\underline{\qquad} \ kJ} \times 100$$

$$\boxed{\% \ Hydrogen \ consumed = \underline{\qquad}}$$

Therefore, the use of hydrogen fuel to heat the reactants would be a feasible option since it would not use a large amount of the vehicle fuel reserve.

11.3-2 Air-Cooling System for a Fuel Cell

A fuel cell is being cooled with air at a flow rate of $33.3\dfrac{\text{moles air}}{\text{min}}$. The residence time of the air inside the cooling chamber is 3 seconds, which allows the temperatures of the air in the chamber and the outlet air to reach thermal equilibrium and therefore exit at the same temperature.

The fuel cell vehicle is operating during the winter in Houghton, Michigan where the temperature of the surroundings is 24°F.

(a) Determine the rate of heat generation by the fuel cell stack if the heat capacity of the air at constant pressure is $C_p = 29.24\dfrac{\text{J}}{\text{mol}\cdot\text{°C}}$. The following diagram describes the cooling process of the fuel cell. At steady state, the temperature of the air in the cooling chamber is 70°F.
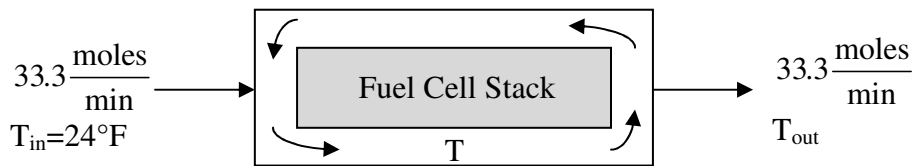
The rate of heat loss from the cooling chamber to its surroundings is given by:

$$\dot{Q}_{lost} = \left(850\dfrac{\text{W}}{\text{°C}}\right)\left(T - T_{in}\right)$$

Strategy

The amount of heat generated by the fuel cell can be obtained by performing an energy balance around the cooling chamber at steady state.

Solution

We can start by writing the general energy balance equation applied to an open system:

$$NC_v\dfrac{dT}{dt} = \dot{n}C_p\left(T_{in} - T\right) + \dot{Q} - W_s$$

The temperature of the air is given in the problem statement when the cooling process is at steady state. Hence, $\dfrac{dT}{dt} = 0$. Substituting this assumption in the energy balance, we have:

$$\dot{n}C_p\left(T_{in} - T\right) + \dot{Q} - \dot{W}_s = 0$$

Daniel López Gaxiola   Student View
Jason M. Keith

Supplemental Material for Elementary Principles of Chemical Processes

where:

$T_{in} = 24°F$

$T = T_{out} = 70°F$

$\dot{W}_s = 0$ (no mechanical parts moving)

$\dot{Q} = \dot{Q}_{gen} - \dot{Q}_{lost}$

It can be seen that the rate of heat loss and the heat capacity at constant pressure are given in $\dfrac{J}{mol \cdot °C}$. Thus, the temperature values, which are given in °F must be converted to °C as follows:

$$T(°C) = \frac{5}{9}[T(°F) - 32]$$

$$T_{in}(°C) = \frac{5}{9}[24°F - 32] = \underline{\hspace{1cm}} °C$$

$$T_{out}(°C) = \frac{5}{9}[70°F - 32] = \underline{\hspace{1cm}} °C$$

These temperatures can be entered into the energy balance equation. After solving for the rate of heat generation by the fuel cell stack, we get:

$$\dot{Q}_{gen} = \left(33.3 \frac{moles}{min}\right)\left(\frac{1\ min}{60\ s}\right)\left(\underline{\hspace{1cm}} \frac{J}{mol \cdot °C}\right)[\underline{\hspace{1cm}} °C - (\underline{\hspace{1cm}} °C)]$$

$$+ \left(850 \frac{W}{°C}\right)[\underline{\hspace{1cm}} °C - (\underline{\hspace{1cm}} °C)]$$

$$\boxed{\dot{Q}_{gen} = \underline{\hspace{1cm}} W}$$

(b) Obtain an equation for the temperature inside the cooling chamber as a function of time.

Strategy

Since we need to get an equation to calculate the temperature as time changes, the accumulation term will have to be included in the energy balance equation for an open system.

Solution

The energy balance equation is given by:

$$NC_v \frac{dT}{dt} = \dot{n}C_p (T_{in} - T) + \dot{Q}_{gen} - \dot{Q}_{lost}$$

where:

N = number of moles of air inside the cooling chamber

C_v = heat capacity of the air at constant volume.

The shaft work has been neglected from this equation since there are no mechanical parts applying work on the system.

The only unknown parameters in the energy balance equation are the number of moles of air N, and the heat capacity at constant volume.

The moles of air inside the system can be calculated by multiplying the residence time θ by the air flow rate:

$$N = \theta\dot{n} = \underline{\quad} s \left(33.3 \frac{mol}{min} \right) \left(\frac{1\ min}{60\ s} \right)$$

$$N = \underline{\quad} \text{ moles}$$

The heat capacity at constant volume can be obtained by subtracting the ideal gas constant to the heat capacity at constant pressure:

$$C_v = C_p - R$$

Substituting the corresponding quantities into this equation yields:

$$C_v = \underline{\quad} \frac{J}{mol \cdot °C} - \underline{\quad} \frac{J}{mol \cdot °C}$$

$$C_v = 20.93 \frac{J}{mol \cdot °C}$$

Daniel López Gaxiola   Student View
Jason M. Keith

Supplemental Material for Elementary Principles of Chemical Processes

Now that all the unknown parameters in the energy balance equation have been calculated, we can substitute the corresponding values into it to get:

$$\left(\underline{\qquad}\ \text{moles}\right)\left(20.93\frac{J}{\text{mol}\cdot°C}\right)\frac{dT}{dt}=\left(33.3\frac{\text{mol}}{\text{min}}\right)\left(\underline{\qquad}\right)\left(29.24\frac{J}{\text{mol}\cdot°C}\right)(\underline{\qquad}°C-T)$$

$$+\underline{\qquad}\ W-\left(850\frac{W}{°C}\right)\left[T-\left(\underline{\quad}°C\right)\right]$$

$$\left(34.85\frac{J}{°C}\right)\frac{dT}{dt}=16.23\frac{W}{°C}(\underline{\qquad}°C-T)+\underline{\qquad}-\left(850\frac{W}{°C}\right)\left[T-\left(\underline{\qquad}°C\right)\right]$$

$$\left(34.85\frac{J}{°C}\right)\frac{dT}{dt}=\left(\underline{\qquad}-16.23T+\underline{\qquad}-850T-\underline{\qquad}\right)W$$

The similar terms in this equation can be reduced to yield:

$$\left(34.85\frac{J}{°C}\right)\frac{dT}{dt}=\left(\underline{\qquad}-866.23T\right)W$$

Solving for the accumulation term, we have:

$$\frac{dT}{dt}=\frac{\left(\underline{\qquad}-866.23T\right)W}{\left(34.85\frac{J}{°C}\right)}=\left(\underline{\qquad}-\underline{\qquad}T\right)\frac{°C}{s}$$

This equation can be now separated and integrated as shown in the following steps. The initial condition for this system is @ t = 0 s, T = -4°C

$$\int_{-4.4°C}^{T}\frac{dT}{\left(\underline{\qquad}-\underline{\qquad}T\right)}=\int_{0}^{t}dt$$

$$-\frac{1}{24.86}\left\{\ln\left[\underline{\qquad}-\underline{\qquad}T\right]-\ln\left[\underline{\qquad}-\underline{\qquad}\left(\underline{\quad}°C\right)\right]\right\}=t$$

This equation can be simplified by applying the logarithm properties:

$$-\frac{1}{24.86}\ln\left[\frac{\underline{\qquad}-\underline{\qquad}T}{\underline{\qquad}-\underline{\qquad}\left(\underline{\quad}°C\right)}\right]=t$$

$$\ln\left[\frac{\underline{\qquad}-\underline{\qquad}T}{\underline{\qquad}+\underline{\qquad}}\right]=-24.86t$$

Daniel López Gaxiola   Student View
Jason M. Keith

Supplemental Material for Elementary Principles of Chemical Processes

$$\frac{\underline{\hspace{1cm}} - \underline{\hspace{1cm}}\,T}{633.84} = \underline{\hspace{1cm}}$$

Solving for the temperature T yields:

$$T = \frac{\underline{\hspace{1cm}} - 633.84\left(\underline{\hspace{1.5cm}}\right)}{24.86}$$

$$\boxed{T = \underline{\hspace{1cm}} - 25.5\left(\underline{\hspace{1.5cm}}\right)}$$

Where t is in seconds and T is in °C.

To verify the validity of this equation, we can evaluate at t = 0 s, to get:

$$T = \underline{\hspace{1cm}} - 25.5\left(\underline{\hspace{1.5cm}}\right)$$

$$T = \underline{\hspace{1cm}} - 25.5\left(\underline{\hspace{1.5cm}}\right)$$

$$T = \underline{\hspace{1cm}}\ °C$$

Since this result agrees with the given initial condition, it is valid to use this equation to estimate the temperature as a function of time t.

In addition, it can be seen that as t ⟶∞, the term with the exponential function in it will approach zero. Thus, the temperature will be given by:

$$T = 21.1 - 25.5(0)$$

$$T = 21.1\,°C$$

This temperature value matches with the steady-state temperature used in part a) to determine the rate of heat generation by the fuel cell.