

Chapter 3
Processes and Process Variables

Name: _____

Date: _____

The goal of Chapter 3 is to introduce physical properties to describe chemical process materials. It is important when solving problems to write down the formula you are using, then rewrite the formula substituting in the proper variables and units. This is not only important when solving the problem but when looking at your solution when studying or designing a similar process when employed in industry or graduate school. The following problems build upon the fundamentals covered in your text as applied to hydrogen processing, hydrogen as an energy carrier, and the use of hydrogen in fuel cells.

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Example 3.1-1 Mass, Volume, and Density

(a) Liquid methanol can be used in portable fuel cell applications for items such as cell phones or laptop computers. Calculate the density of methanol in lb_m/ft^3 from a tabulated specific gravity, and determine the volume in cubic inches needed to contain 100 g of methanol.

Strategy

First, we will determine the specific gravity of methanol from Table B.1 of the textbook and use it to determine the density. Once the density is determined, unit conversions can be used to determine the required volume.

Solution

Table B.1 lists the specific gravity of methanol as 0.792. Thus, the density is given as:

$$\rho_{\text{CH}_3\text{OH}} = (0.792) \left(62.43 \frac{\text{lb}_m}{\text{ft}^3} \right) = \frac{\text{lb}_m}{\text{ft}^3}$$

The volume is calculated from the mass divided by the density. Thus,

$$V = \frac{100 \text{ g}}{454 \text{ g}} \frac{1 \text{ lb}_m}{\text{lb}_m} \frac{\text{ft}^3}{\text{ft}^3} \frac{\text{in}^3}{\text{in}^3}$$

$$\boxed{V = \text{in}^3}$$

(b) Liquid butane can be used in portable fuel cell applications for items such as cell phones or laptop computers. If the density of butane is $37.5 \frac{\text{lb}_m}{\text{ft}^3}$ determine its specific gravity and determine the volume in cubic inches needed to contain 220 g of butane.

Strategy

First, we will determine the specific gravity of butane from the given and reference densities. Once the density is determined, unit conversions can be used to determine the required volume.

Solution

The specific gravity is given as:

$$SG = \frac{\rho_{\text{butane}}}{\rho_{\text{H}_2\text{O}}} = \frac{37.5 \frac{\text{lb}_m}{\text{ft}^3}}{\frac{\text{lb}_m}{\text{ft}^3}}$$

$$\boxed{SG = \underline{\quad}}$$

The volume is calculated from the mass divided by the density. Thus,

$$V = \frac{220 \text{ g}}{\text{g}} \frac{1 \text{ lb}_m}{\text{lb}_m} \frac{\text{ft}^3}{\text{ft}^3} \frac{12^3 \text{ in}^3}{\text{ft}^3}$$

$$\boxed{V = \underline{\quad} \text{ in}^3}$$

Example 3.1-2 Effect of Temperature on Liquid Density

Liquid methanol can be used in portable fuel cell applications for items such as cell phones or laptop computers.

Table 3-147 of Perry's Chemical Engineers' Handbook 6th Edition lists the volume expansion of methanol with temperature in degrees Celsius according to:

$$\frac{V(T)}{V_0} = [1 + 1.1342 \times 10^{-3} T + 1.3635 \times 10^{-6} T^2 + 0.8741 \times 10^{-8} T^3]$$

where V_0 is the volume at 0°C.

(a) Determine the fractional increase in volume when the temperature changes from 20°C to a temperature of 40°C.

Strategy

We can apply the given correlation to determine the fractional increase $\frac{V(T)}{V_0}$.

Solution

Using the correlation we find at 20°C and 40°C, respectively:

$$\frac{V(20)}{V_0} = [1 + 1.1342 \times 10^{-3} (\quad) + 1.3635 \times 10^{-6} (\quad)^2 + 0.8741 \times 10^{-8} (\quad)^3]$$

such that

$$V(20) = \quad V_0$$

Also,

$$\frac{V(40)}{V_0} = [1 + 1.1342 \times 10^{-3} (\quad) + 1.3635 \times 10^{-6} (\quad)^2 + 0.8741 \times 10^{-8} (\quad)^3]$$

such that

$$V(40) = \quad V_0$$

Thus,

$$\boxed{\frac{V(40)}{V(20)} = \underline{\hspace{2cm}}}$$

(b) Although the correlation is only valid above 0°C, estimate the fractional increase in volume when the temperature changes from 20°C to a temperature of -20°C.

Strategy

We can apply the given correlation to determine the fractional increase $\frac{V(T)}{V_0}$.

Solution

Using the correlation we find at -20°C:

$$\frac{V(-20)}{V_0} = [1 + 1.1342 \times 10^{-3}(\underline{\hspace{1cm}}) + 1.3635 \times 10^{-6}(\underline{\hspace{1cm}})^2 + 0.8741 \times 10^{-8}(\underline{\hspace{1cm}})^3]$$

such that

$$V(-20) = \underline{\hspace{1cm}} V_0$$

Thus,

$$\boxed{\frac{V(-20)}{V(20)} = \underline{\hspace{2cm}}}$$

Example 3.3-1 Conversion Between Mass and Moles

One possible short-term source of hydrogen for fuel cells is from coal.

(a) Consider a metric ton (1000 kg) of coal which contains 530 kg carbon, 190 kg oxygen, and 50 kg hydrogen with the rest impurities. Determine the number of moles of carbon, oxygen, and hydrogen in the metric ton of coal.

Strategy

We can use molecular weight formulas and unit conversions to determine the number of moles.

Solution

For carbon, we have:

$$\frac{530 \text{ kg C}}{\text{kg C}} \frac{1000 \text{ g C}}{\text{g C}} \frac{\text{mol C}}{\text{g C}} = \text{_____ mol C}$$

For oxygen, we have:

$$\frac{190 \text{ kg O}}{\text{kg O}} \frac{1000 \text{ g O}}{\text{g O}} \frac{\text{mol O}}{\text{g O}} = \text{_____ mol O}$$

For hydrogen, we have:

$$\frac{\text{_____ kg H}}{\text{kg H}} \frac{1000 \text{ g H}}{\text{g H}} \frac{\text{mol H}}{\text{g H}} = \text{_____ mol H}$$

(b) Consider a coal pile of the same composition as in problem a above. If there is 1.0×10^6 mol C, determine the kg C and H in the coal pile.

Strategy

We can use molecular weight formulas and unit conversions to work backwards to determine the masses of carbon and hydrogen.

Solution

For carbon, we have:

$$\frac{\text{_____ mol C}}{\text{mol C}} \frac{\text{g C}}{\text{g C}} \frac{\text{kg C}}{1000 \text{ g C}} = \text{_____ kg C}$$

For hydrogen, we can apply the ratio to give:

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$$\frac{\text{_____ kg C}}{\text{_____ kg C}} \frac{50 \text{ kg H}}{\text{_____ kg C}} = \text{_____ kg H}$$

Example 3.3-2 Conversions Using Mass and Mole Fractions

Assume that a stream exiting a fuel cell contains 20 mol % H₂O and 85 mass % N₂. Determine the following quantities:

- (a) Moles of H₂O in 100 mol of gas
- (b) Mass of N₂ in 100 g of gas
- (c) Mass of H₂O in 100 mol of gas
- (d) Moles of N₂ in 100 mol of gas

Strategy

We can use the mass and mole fraction values along with molecular weights to determine the required quantities.

Solution

- (a) Using the mole fraction formula, we have:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{mol H}_2\text{O}}{1 \text{ mol gas}} = \boxed{\text{mol H}_2\text{O}}$$

- (b) Using the mass fraction formula, we have:

$$\frac{100 \text{ g gas}}{1 \text{ g gas}} \frac{\text{g N}_2}{1 \text{ g gas}} = \boxed{\text{g N}_2}$$

- (c) Using the mole fraction formula and the molecular weight, we have:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{mol H}_2\text{O}}{1 \text{ mol gas}} \frac{\text{g H}_2\text{O}}{1 \text{ mol H}_2\text{O}} = \boxed{\text{g H}_2\text{O}}$$

- (d) The mole fraction of N₂ can be determined as $1 - x_{\text{H}_2\text{O}} = \text{_____}$. Thus, using the mole fraction formula we have:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{mol N}_2}{1 \text{ mol gas}} = \boxed{\text{mol H}_2\text{O}}$$

Example 3.3-3 Conversion from a Composition by Mass to a Molar Composition

(a) A stream exiting a fuel cell contains 21 mol% H₂O, and the balance N₂. Determine the mass fraction of all gases in the exit stream.

Strategy

We will have to choose an arbitrary basis to solve this problem. We will use 100 mol. Then we can use the molecular weights to determine the mass in grams. The sum of the individual masses is the total mass. The mass fraction of each gas is then determined by dividing the individual mass by the total mass.

Solution

The mass of water is given by:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{21 \text{ mol H}_2\text{O}}{1 \text{ mol H}_2\text{O}} \frac{18 \text{ g H}_2\text{O}}{1 \text{ mol H}_2\text{O}} = 378 \text{ g H}_2\text{O}$$

Noting that the mole fractions must sum to unity, the nitrogen mole fraction is 0.79. Thus, the mass of nitrogen is given by:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{79 \text{ mol N}_2}{1 \text{ mol N}_2} \frac{28 \text{ g N}_2}{1 \text{ mol N}_2} = 2212 \text{ g N}_2$$

The total mass is the sum of the individual masses and is 2590 g. The mass fractions are determined by dividing the individual masses by the total. Thus, for water we have:

$$\frac{378 \text{ g H}_2\text{O}}{2590 \text{ g gas}} = \boxed{0.146 \text{ mole frac H}_2\text{O}}$$

And for nitrogen we have:

$$\frac{2212 \text{ g N}_2}{2590 \text{ g gas}} = \boxed{0.854 \text{ mole frac N}_2}$$

(b) A stream exiting a fuel cell contains 5.7 mol% H₂, 8.6 mol% O₂, 17.1 mol% H₂O, and the balance N₂. Determine the mass fraction of all gases in the exit stream.

Strategy

We will have to choose an arbitrary basis to solve this problem. We will use 100 mol. Then we can use the molecular weights to determine the mass in grams. The sum of the individual

masses is the total mass. The mass fraction of each gas is then determined by dividing the individual mass by the total mass.

Solution

The mass of hydrogen is given by:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{mol H}_2}{\text{mol H}_2} \frac{2 \text{ g H}_2}{\text{mol H}_2} = \text{g H}_2$$

The mass of oxygen is given by:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{mol O}_2}{\text{mol O}_2} \frac{\text{g O}_2}{\text{mol O}_2} = \text{g O}_2$$

The mass of water is given by:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{mol H}_2\text{O}}{\text{mol H}_2\text{O}} \frac{\text{g H}_2\text{O}}{\text{mol H}_2\text{O}} = \text{g H}_2\text{O}$$

Noting that the mole fractions must sum to unity, the nitrogen mole fraction is 0.686. Thus, the mass of nitrogen is given by:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{0.686 \text{ mol N}_2}{\text{mol N}_2} \frac{\text{g N}_2}{\text{mol N}_2} = \text{g N}_2$$

The total mass is the sum of the individual masses and is _____ g. The mass fractions are determined by dividing the individual masses by the total. Thus, for hydrogen we have:

$$\frac{11.4 \text{ g H}_2}{\text{g gas}} = \boxed{\text{mole frac H}_2}$$

Similarly, for oxygen we have:

$$\frac{\text{g O}_2}{\text{g gas}} = \boxed{\text{mole frac O}_2}$$

For water we have:

$$\frac{\text{g H}_2\text{O}}{\text{g gas}} = \boxed{\text{mole frac H}_2\text{O}}$$

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Finally, for nitrogen we have:

$$\frac{\text{g N}_2}{\text{g gas}} = \boxed{0.764 \text{ mole frac N}_2}$$

Example 3.3-4 Calculation of an Average Molecular Weight

(a) Determine the average molecular weight of a stream exiting a fuel cell containing 10 mol% water, 10 mol% oxygen, and 80 mol% nitrogen.

Strategy

First, we can choose an arbitrary basis for the problem, perhaps 100 mol. Then we can use the mole fraction and molecular weights to determine the mass of each gas in the stream. The masses can be summed and divided by the basis to determine the average molecular weight.

Solution

The mass of water in the stream is:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{0.10 \text{ mol H}_2\text{O}}{1 \text{ mol H}_2\text{O}} \frac{18 \text{ g H}_2\text{O}}{1 \text{ mol H}_2\text{O}} = \text{---} \text{ g H}_2\text{O}$$

Similarly, the mass of oxygen in the stream is:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{---} \text{ mol O}_2}{1 \text{ mol O}_2} \frac{\text{---} \text{ g O}_2}{1 \text{ mol O}_2} = 320 \text{ g O}_2$$

Finally, the mass of nitrogen in the stream is:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{---} \text{ mol H}_2\text{O}}{1 \text{ mol H}_2\text{O}} \frac{\text{---} \text{ g N}_2}{1 \text{ mol N}_2} = \text{---} \text{ g N}_2$$

The total mass is _____ g. Dividing by the basis of 100 mol, the average molecular weight of the gas is $\boxed{\frac{\text{---} \text{ g}}{\text{---} \text{ mol}}}$.

(b) Determine the average molecular weight of a stream exiting a natural gas fuel reformer containing 15.7 mol% methane, 10.7 mol% water, 5.3 mol% carbon dioxide, 12.3 mol% carbon monoxide, and 56.0 mol% hydrogen.

Strategy

First, we can choose an arbitrary basis for the problem, perhaps 100 mol. Then we can use the mole fraction and molecular weights to determine the mass of each gas in the stream. The masses can be summed and divided by the basis to determine the average molecular weight.

Solution

The mass of methane in the stream is:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{0.157 \text{ mol CH}_4}{1 \text{ mol CH}_4} \frac{16 \text{ g CH}_4}{1 \text{ mol CH}_4} = \text{_____ g CH}_4$$

The mass of water in the stream is:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{0.107 \text{ mol H}_2\text{O}}{1 \text{ mol H}_2\text{O}} \frac{\text{___ g H}_2\text{O}}{1 \text{ mol H}_2\text{O}} = \text{_____ g H}_2\text{O}$$

Similarly, the mass of carbon dioxide in the stream is:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{_____ mol CO}_2}{1 \text{ mol CO}_2} \frac{\text{___ g CO}_2}{1 \text{ mol CO}_2} = \text{_____ g CO}_2$$

Similarly, the mass of carbon monoxide in the stream is:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{_____ mol CO}}{1 \text{ mol CO}} \frac{\text{___ g CO}}{1 \text{ mol CO}} = \text{_____ g CO}$$

Finally, the mass of nitrogen in the stream is:

$$\frac{100 \text{ mol gas}}{1 \text{ mol gas}} \frac{\text{_____ mol N}_2}{1 \text{ mol N}_2} \frac{\text{___ g N}_2}{1 \text{ mol N}_2} = \text{_____ g N}_2$$

The total mass is _____ g. Dividing by the basis of 100 mol, the average molecular weight of

the gas is $\boxed{\frac{\text{_____ g}}{\text{_____ mol}}}$.

Example 3.3-5 Conversion between Mass, Molar, and Volumetric Flow Rates of a Gas

(a) You have a fuel cell being fed by 0.5 standard liters per minute (SLPM) hydrogen. Determine the mass and molar flow rates in g/s and mol/s, respectively.

Strategy

We can use unit conversions and the ideal gas law to make the proper conversions. We will convert to mol/s first.

Solution

Noting that standard temperature and pressure are 273 K and 1 atm, respectively, we can use the ideal gas law:

$$P\dot{V} = \dot{n}RT$$

Solving for the molar flow rate \dot{n} and substituting the values into this equation gives:

$$\dot{n} = \frac{P\dot{V}}{RT}$$

$$\dot{n} = \frac{1 \text{ atm} \frac{\text{L}}{\text{min}}}{0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \frac{\text{K}}{1} \frac{1 \text{ min}}{\text{s}}}$$

$$\dot{n} = \frac{\text{mol}}{\text{s}}$$

We can then use the molecular weight to determine the mass flow rate as:

$$\frac{\text{mol}}{\text{s}} \frac{2 \text{ g}}{\text{mol}} = \frac{\text{g}}{\text{s}}$$

(b) You have a fuel cell being fed by 2.0 mol/hr hydrogen. Determine the mass and molar flow rates in g/s and mol/s, respectively. Also determine the volumetric flow rate in SLPM.

Strategy

We can use unit conversions and the ideal gas law to make the proper conversions. We will convert to mol/s first.

Solution

We first convert the time units on the molar flow rate from hours to seconds. This gives:

$$\frac{2 \text{ mol}}{\text{hr}} \frac{\text{hr}}{3600 \text{ s}} = \boxed{\frac{\text{mol}}{\text{s}}}$$

Using the molecular weight we have the mass flow rate:

$$\frac{\text{mol}}{\text{s}} \frac{2 \text{ g}}{\text{mol}} = \boxed{\frac{\text{g}}{\text{s}}}$$

Noting that standard temperature and pressure are 273 K and 1 atm, respectively, we can use the ideal gas law to give:

$$\frac{\text{mol}}{\text{s}} \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \frac{\text{K}}{1 \text{ atm}} \frac{60 \text{ s}}{1 \text{ min}} = \boxed{\text{SLPM}}$$

Example 3.4-1 Calculation of a Pressure as a Head of Fluid

(a) A compressor delivers air at 3.0 atm to a fuel cell. Express the pressure in inches of liquid mercury.

Strategy

We can use the density of mercury and unit conversions to give an equivalent head of mercury to produce the prescribed pressure.

Solution

Simply, we can use unit conversions to give:

$$\frac{3 \text{ atm}}{1 \text{ atm}} \frac{\text{in Hg}}{1 \text{ atm}} = \text{___ in Hg}$$

Alternatively, we can use the relationship $P = \rho g P_h$ to determine the pressure head P_h . To do so, we need the specific gravity of liquid mercury, which is 13.6. Thus, the density is equal to $13.6 \times 1000 \text{ kg/m}^3 = 13600 \text{ kg/m}^3$.

As such, we have $P_h = \frac{P}{\rho g}$. Then, using unit conversions, we have:

$$P_h = \frac{P}{\rho g} = \frac{\text{___ atm}}{1 \text{ atm}} \frac{\frac{\text{N}}{\text{m}^2} \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{\text{N}} \frac{\text{m}^3}{13600 \text{ kg}} \frac{\text{s}^2}{9.8 \text{ m}} \frac{\text{in}}{\text{m}}$$

$$\boxed{P_h = \text{___ in}}$$

(b) A compressor delivers air at 2.0 atm to a fuel cell. Express the pressure in feet of liquid water.

Strategy

We can use the density of mercury and unit conversions to give an equivalent head of water to produce the prescribed pressure.

Solution

Simply, we can use unit conversions to give:

$$\frac{2 \text{ atm}}{1 \text{ atm}} \frac{33.9 \text{ ft H}_2\text{O}}{1 \text{ atm}} = \text{___ ft H}_2\text{O}$$

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Alternatively, we can use the relationship $P = \rho g P_h$ to determine the pressure head P_h . To do so, we need the density of liquid water, equal to 1000 kg/m^3 .

As such, we have $P_h = \frac{P}{\rho g}$. Then, using unit conversions, we have:

$$P_h = \frac{P}{\rho g} = \frac{\text{--- atm}}{1 \text{ atm}} \frac{\text{---} \frac{\text{N}}{\text{m}^2}}{\text{---} \frac{\text{kg} \cdot \text{m}}{\text{s}^2}} \frac{\text{---} \text{m}^3}{\text{---} \text{kg}} \frac{\text{---} \text{s}^2}{9.8 \text{ m}} \frac{\text{---} \text{ft}}{\text{---} \text{m}}$$

$$\boxed{P_h = \text{--- ft H}_2\text{O}}$$

Example 3.4-3 Pressure Measurement with Manometers

A differential manometer is being used to measure the pressure drop of hydrogen in a gas manifold. The manifold supplies hydrogen fuel at 20°C via parallel channels to several fuel cells connected electrically in series.

(a) The manometer fluid is water at 20°C. The measured level in the left and right arms of the manometer are 85 and 197 mm, respectively.

Estimate the pressure drop in psi.

Strategy

We use the differential manometer equation to estimate the pressure drop.

Solution

Using the differential manometer equation, we have:

$$P_2 - P_1 = (\rho_f - \rho)gh$$

Since the gas density is significantly smaller than the liquid density, we can rewrite the above equation as:

$$P_2 - P_1 = \rho_f gh$$

Thus,

$$P_2 - P_1 = \frac{1000 \text{ kg}}{\text{m}^3} \frac{9.8 \text{ m}}{\text{s}^2} \frac{\text{--- mm} - \text{--- mm}}{1000 \text{ mm}} \frac{\text{m}}{\text{kg} \cdot \text{m}} \frac{\text{N}}{\text{s}^2} \frac{\text{--- psi}}{\frac{\text{N}}{\text{m}^2}}$$

$$\boxed{P_2 - P_1 = \text{--- psi}}$$

(b) The manometer fluid is liquid mercury at 20°C. The measured level in the left and right arms of the manometer are 28 and 40 mm, respectively.

Estimate the pressure drop in psi.

Strategy

We use the differential manometer equation to estimate the pressure drop.

Solution

Using the differential manometer equation, we have:

$$P_2 - P_1 = (\rho_f - \rho)gh$$

Since the gas density is significantly smaller than the liquid density, we can rewrite the above equation as:

$$P_2 - P_1 = \rho_f gh$$

Since the specific gravity of liquid mercury is 13.6, we have:

$$P_2 - P_1 = \frac{\text{---} \times 1000 \text{ kg}}{\text{m}^3} \frac{9.8 \text{ m}}{\text{s}^2} \frac{\text{---} \text{ mm} - \text{---} \text{ mm}}{1000 \text{ mm}} \frac{\text{m}}{\text{---}} \frac{\text{N}}{\text{s}^2} \frac{\text{---} \text{ psi}}{\text{---} \frac{\text{N}}{\text{m}^2}}$$

$P_2 - P_1 = \text{---} \text{ psi}$

Example 3.5-2 Temperature Conversion

(a) A proton exchange membrane fuel cell operates at 80.0°C. Determine the temperature in degrees Kelvin, Fahrenheit, and Rankine.

Strategy

We can use temperature conversion formulas to solve this problem.

Solution

Using the temperature conversion formulas we have:

$$T(\text{K}) = T(^{\circ}\text{C}) + 273.15 = \underline{\hspace{2cm}} + 273.15$$

$$\boxed{T(\text{K}) = \underline{\hspace{2cm}} \text{ K}}$$

$$T(^{\circ}\text{F}) = 1.8T(^{\circ}\text{C}) + 32 = 1.8(\underline{\hspace{2cm}}) + 32$$

$$\boxed{T(^{\circ}\text{F}) = \underline{\hspace{2cm}} ^{\circ}\text{F}}$$

$$T(\text{R}) = T(^{\circ}\text{F}) + 459.67 = \underline{\hspace{2cm}} + 459.67$$

$$\boxed{T(\text{R}) = \underline{\hspace{2cm}} \text{ R}}$$

(b) A solid oxide fuel cell operates at 1200°F. Determine the temperature in degrees Kelvin, Celsius, and Rankine.

Strategy

We can use temperature conversion formulas to solve this problem.

Solution

Using the temperature conversion formulas we have:

$$T(\text{R}) = T(^{\circ}\text{F}) + \underline{\hspace{2cm}} = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$\boxed{T(\text{R}) = \underline{\hspace{2cm}} \text{ R}}$$

$$T(^{\circ}\text{C}) = \frac{T(^{\circ}\text{F}) - \underline{\hspace{2cm}}}{\underline{\hspace{2cm}}} = \frac{\underline{\hspace{2cm}} - \underline{\hspace{2cm}}}{\underline{\hspace{2cm}}}$$

$$\boxed{T(^{\circ}\text{C}) = \underline{\hspace{2cm}} ^{\circ}\text{C}}$$

$$T(\text{K}) = T(^{\circ}\text{C}) + \underline{\hspace{2cm}} = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$\boxed{T(\text{K}) = \underline{\hspace{2cm}} \text{ K}}$$

Example 3.5-3 Temperature Conversion and Dimensional Homogeneity

The heat capacity of hydrogen gas in units of $\frac{\text{kJ}}{\text{mol} \cdot ^\circ\text{C}}$ is approximately given by the following relationship, where T is the temperature measured in $^\circ\text{C}$.

$$C_p \left(\frac{\text{kJ}}{\text{mol} \cdot ^\circ\text{C}} \right) = 0.02884 + 7.65 \times 10^{-8} T(^{\circ}\text{C})$$

(a) Derive a formula for heat capacity in units of $\frac{\text{BTU}}{\text{lb}_m \cdot ^\circ\text{F}}$.

Strategy

We can use unit conversions, molecular weight, and temperature conversion formulas to solve this problem.

Solution

We can substitute the relationship $T(^{\circ}\text{C}) = \frac{T(^{\circ}\text{F}) - 32}{1.8}$ into the above equation. We can also use the molecular weight with energy and mass unit conversions to give:

$$C_p = \left[0.02884 + 7.65 \times 10^{-8} \frac{T(^{\circ}\text{F}) - 32}{1.8} \right] \frac{\text{kJ}}{\text{mol} \cdot ^\circ\text{C}} \frac{\text{Btu}}{\text{kJ}} \frac{^\circ\text{C}}{1.8^{\circ}\text{F}} \frac{\text{mol}}{2.016\text{g}} \frac{\text{g}}{\text{lb}_m}$$

$$C_p = \left[0.02884 + 7.65 \times 10^{-8} \frac{T(^{\circ}\text{F}) - 32}{1.8} \right] \frac{\text{Btu}}{\text{lb}_m \cdot ^\circ\text{F}} \left(\text{ ______ } \right)$$

$$C_p = \left[\text{ ______ } + \text{ ______ } T(^{\circ}\text{F}) \right] \frac{\text{Btu}}{\text{lb}_m \cdot ^\circ\text{F}}$$

(b) Derive a formula for heat capacity in units of $\frac{\text{BTU}}{\text{lb}_m \cdot \text{R}}$.

Strategy

We can use the answer for part (a) to derive the new formula.

Solution

We can substitute the relationship $T(^{\circ}\text{F}) = T(\text{R}) - 459.67$ into the above equation. This gives:

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$$C_p = [3.4197 + 5.039 \times 10^{-6} T(^{\circ}\text{F})] \frac{\text{Btu}}{\text{lb}_m \cdot ^{\circ}\text{F}}$$

$$C_p = [3.4197 + 5.039 \times 10^{-6} [T(\text{R}) - \text{_____}]] \frac{\text{Btu}}{\text{lb}_m \cdot ^{\circ}\text{F}} \frac{^{\circ}\text{F}}{\text{R}}$$

$$\boxed{C_p = [\text{_____} + \text{_____} T(\text{R})] \frac{\text{Btu}}{\text{lb}_m \cdot \text{R}}}$$

Note that the values are the same since ΔT in R is the same as ΔT in $^{\circ}\text{F}$.