**CACHE Modules on Energy in the Curriculum**

**Module Title:** Power and Inverters

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**Introduction**: Use of the right type of inverter is an essential factor for increasing the efficiency of an array of solar panels. In almost all cases, the actual output of a panel is at least slightly different than the rated ideal output. Shading, changing light intensity and small manufacturing errors cause variation in the efficiency of a panel and in turn in the output of the array. Hence some inverters have a maximum power point tracking (MPPT) function which enables them to increase the output. The common types of inverters used are centralized String inverters and multiple Micro-inverters. In this module students will analyze the functioning of these inverters and the corresponding output they would provide. Students are advised to refer the module titled Fill Factor of Solar Cells for better understanding of this module. Use of MS Excel would be recommended.

**Example Problems:**

1) Plot an I-V curve for a Silicon solar cell with Voc = 40 V and Isc = 4 A and determine the maximum power point. Refer to the module titled Fill Factor for Solar Cells. Use m=8.8 and n=0.75.

Solution:

We have studied in the module Fill Factor of Solar cells that the current and voltage are related by the relationship vm + jn = 1 where v = V/Voc and j = J/Jsc such that v and j lie between 0 and 1. We can plot a graph of j vs v by assuming values in this range for v and calculating values for j using the equation 

The following table provides the values used for plotting the j vs v graph

|  |  |
| --- | --- |
| v |  |
| 0 | 1.000000 |
| 0.1 | 1.000000 |
| 0.2 | 0.999999 |
| 0.3 | 0.999967 |
| 0.4 | 0.999580 |
| 0.5 | 0.997010 |
| 0.6 | 0.985145 |
| 0.7 | 0.942638 |
| 0.8 | 0.817398 |
| 0.9 | 0.510934 |
| 1 | 0.000000 |

Since we know that v = V/Voc and j = J/Jsc we can obtain the values of I and V for plotting a I-V curve by multiplying v by Voc = 40V and by multiplying j by Isc = 4A. Note that even though j is a ratio of current densities, it is as good as taking the ratio of current since the units of area cancel out.

The values of I and V for the I-V curve are given in the table below

|  |  |
| --- | --- |
| V | I |
| 0 | 4.000000 |
| 4 | 4.000000 |
| 8 | 3.999996 |
| 12 | 3.999866 |
| 16 | 3.998321 |
| 20 | 3.988039 |
| 24 | 3.940582 |
| 28 | 3.770553 |
| 32 | 3.269590 |
| 36 | 2.043736 |
| 40 | 0.000000 |

2) Consider an array of 10 panels of crystalline silicon solar cells. The open circuit voltage in Volts and short circuit current in Amperes for every panel is given in the table below.

|  |  |  |
| --- | --- | --- |
| Panel# | Voc | Isc |
| 1 | 44.83 | 5.75 |
| 2 | 42.50 | 5.30 |
| 3 | 44.30 | 5.60 |
| 4 | 43.40 | 5.70 |
| 5 | 44.80 | 5.50 |
| 6 | 43.60 | 5.70 |
| 7 | 42.90 | 5.50 |
| 8 | 43.00 | 5.65 |
| 9 | 42.00 | 5.20 |
| 10 | 42.50 | 5.70 |

a) Plot a graph showing the I-V curve of each panel assuming m=8.8 and n=0.75.

b) Estimate the maximum power point for each panel by using the values for the I-V graph.

c) Find the effective power of the array if following types of inverter are used:

 i) String inverter

ii) String inverter with MPPT and

iii) Micro-inverter for every panel.

Solution:

a) The I-V curve for each panel can be separately calculated using the method used in Problem 1 and then a merged graph can be prepared.

Thus the values of V and I for every panel would be:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Panel 1 | Panel 2 | Panel 3 | Panel 4 | Panel 5 | Panel 6 | Panel 7 | Panel 8 | Panel 9 | Panel 10 |
|
| V | I | V | I | V | I | V | I | V | I | V | I | V | I | V | I | V | I | V | I |
| 0.00 | 5.75 | 0.00 | 5.30 | 0.00 | 5.60 | 0.00 | 5.70 | 0.00 | 5.50 | 0.00 | 5.70 | 0.00 | 5.50 | 0.00 | 5.65 | 0.00 | 5.20 | 0.00 | 5.70 |
| 4.48 | 5.75 | 4.25 | 5.30 | 4.43 | 5.60 | 4.34 | 5.70 | 4.48 | 5.50 | 4.36 | 5.70 | 4.29 | 5.50 | 4.30 | 5.65 | 4.20 | 5.20 | 4.25 | 5.70 |
| 8.97 | 5.75 | 8.50 | 5.30 | 8.86 | 5.60 | 8.68 | 5.70 | 8.96 | 5.50 | 8.72 | 5.70 | 8.58 | 5.50 | 8.60 | 5.65 | 8.40 | 5.20 | 8.50 | 5.70 |
| 13.45 | 5.75 | 12.75 | 5.30 | 13.29 | 5.60 | 13.02 | 5.70 | 13.44 | 5.50 | 13.08 | 5.70 | 12.87 | 5.50 | 12.90 | 5.65 | 12.60 | 5.20 | 12.75 | 5.70 |
| 17.93 | 5.75 | 17.00 | 5.30 | 17.72 | 5.60 | 17.36 | 5.70 | 17.92 | 5.50 | 17.44 | 5.70 | 17.16 | 5.50 | 17.20 | 5.65 | 16.80 | 5.20 | 17.00 | 5.70 |
| 22.42 | 5.73 | 21.25 | 5.28 | 22.15 | 5.58 | 21.70 | 5.68 | 22.40 | 5.48 | 21.80 | 5.68 | 21.45 | 5.48 | 21.50 | 5.63 | 21.00 | 5.18 | 21.25 | 5.68 |
| 26.90 | 5.66 | 25.50 | 5.22 | 26.58 | 5.52 | 26.04 | 5.62 | 26.88 | 5.42 | 26.16 | 5.62 | 25.74 | 5.42 | 25.80 | 5.57 | 25.20 | 5.12 | 25.50 | 5.62 |
| 31.38 | 5.42 | 29.75 | 5.00 | 31.01 | 5.28 | 30.38 | 5.37 | 31.36 | 5.18 | 30.52 | 5.37 | 30.03 | 5.18 | 30.10 | 5.33 | 29.40 | 4.90 | 29.75 | 5.37 |
| 35.86 | 4.70 | 34.00 | 4.33 | 35.44 | 4.58 | 34.72 | 4.66 | 35.84 | 4.50 | 34.88 | 4.66 | 34.32 | 4.50 | 34.40 | 4.62 | 33.60 | 4.25 | 34.00 | 4.66 |
| 40.35 | 2.94 | 38.25 | 2.71 | 39.87 | 2.86 | 39.06 | 2.91 | 40.32 | 2.81 | 39.24 | 2.91 | 38.61 | 2.81 | 38.70 | 2.89 | 37.80 | 2.66 | 38.25 | 2.91 |
| 44.83 | 0.00 | 42.50 | 0.00 | 44.30 | 0.00 | 43.40 | 0.00 | 44.80 | 0.00 | 43.60 | 0.00 | 42.90 | 0.00 | 43.00 | 0.00 | 42.00 | 0.00 | 42.50 | 0.00 |

These data were obtained by multiplying Voc and Isc of each panel with the values of v and j calculated at the start of the example problem solution.

The graph with I-V curves of all the panels would be

b) Using this graph we can determine the maximum power point for each and every panel in the array. However since the graph is unclear we can use the alternative method wherein which we would calculate the power for every value of V and I of every panel using the equation P=V\*I. This power in Watts for each panel can be tabulated as

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Panel 1 | Panel 2 | Panel 3 | Panel 4 | Panel 5  | Panel 6 | Panel 7 | Panel 8 | Panel 9 | Panel 10 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 25.77 | 22.52 | 24.81 | 24.74 | 24.64 | 24.85 | 23.59 | 24.29 | 21.84 | 24.22 |
| 51.55 | 45.05 | 49.62 | 49.48 | 49.28 | 49.70 | 47.19 | 48.59 | 43.68 | 48.45 |
| 77.32 | 67.57 | 74.42 | 74.21 | 73.92 | 74.55 | 70.78 | 72.88 | 65.52 | 72.67 |
| 103.05 | 90.06 | 99.19 | 98.91 | 98.52 | 99.37 | 94.34 | 97.14 | 87.32 | 96.86 |
| 128.48 | 112.29 | 123.67 | 123.32 | 122.83 | 123.89 | 117.62 | 121.11 | 108.87 | 120.76 |
| 152.34 | 133.14 | 146.64 | 146.22 | 145.64 | 146.90 | 139.47 | 143.60 | 129.09 | 143.19 |
| 170.06 | 148.63 | 163.69 | 163.23 | 162.59 | 163.99 | 155.69 | 160.31 | 144.11 | 159.85 |
| 168.53 | 147.30 | 162.22 | 161.77 | 161.13 | 162.51 | 154.29 | 158.87 | 142.82 | 158.41 |
| 118.51 | 103.58 | 114.08 | 113.76 | 113.30 | 114.28 | 108.50 | 111.72 | 100.43 | 111.40 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

From these values we can identify the exact maximum power of every panel in the array. The values for each panel in Watts would be:

|  |  |
| --- | --- |
| Panel# | Pmax |
| 1 | 170.06 |
| 2 | 148.63 |
| 3 | 163.69 |
| 4 | 163.23 |
| 5 | 162.59 |
| 6 | 163.99 |
| 7 | 155.69 |
| 8 | 160.31 |
| 9 | 144.11 |
| 10 | 159.85 |

It is noted that since m and n are the same for all these panels in this example problem (they may not be in practice), the maximum power will occur when v = 0.8.

c) An ideal array would be the one in which every panels work at their maximum power point. The ability to get all the panels working at their maximum power point is different for different types of inverters. Hence, the effective power of an array varies with the type of inverters used.

i) String Inverter: A String Inverter is a centralized inverter and the panels are connected in series to it. Each panel would perform according to the conditions of the poorest performer (lowest maximum power) in the array. In our case the lowest maximum power is for panel 9 which is 144.11 W and the voltage at that point is 29.40 V. We would have to find out the current for every panel in the array when the voltage across the array is 29.40 V and consequently the power of every panel at that voltage.

Let us consider Panel 2,

We know that V = 29.40 V and Voc = 42.5 V.

$$v=\frac{V}{V\_{oc}}=\frac{29.40}{42.5}=0.69$$

$$j=\left(1-v^{m}\right)^{\frac{1}{n}}=\left(1-0.69^{8.8}\right)^{\frac{1}{0.75}}=0.95$$

But j = J/Jsc = I/Isc

$$I=j×I\_{sc}=0.95×5.3=5.03 A$$

$$P=V×I=29.40×5.03=147.88 W$$

This procedure is followed for all of the panels and the values of v, j, I (in Amperes) and P (in Watts) at 29.40 V for every panel are given below. The total effective power of the array would be the sum of the power of each panel.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Panel# | v | J | I | P |
| 1 | 0.66 | 0.97 | 5.56 | 163.54 |
| 2 | 0.69 | 0.95 | 5.03 | 147.76 |
| 3 | 0.66 | 0.96 | 5.40 | 158.72 |
| 4 | 0.68 | 0.96 | 5.45 | 160.36 |
| 5 | 0.66 | 0.97 | 5.32 | 156.43 |
| 6 | 0.67 | 0.96 | 5.46 | 160.65 |
| 7 | 0.69 | 0.95 | 5.24 | 153.99 |
| 8 | 0.68 | 0.95 | 5.39 | 158.35 |
| 9 | 0.70 | 0.94 | 4.90 | 144.11 |
| 10 | 0.69 | 0.95 | 5.41 | 158.91 |
| Effective Power |  |  | 1562.82 |

Thus, the array has a total effective power of 1562.824 W is a String Inverter is used.

ii) String Inverter with MPPT: A String inverter with MPPT (Maximum Power Point Tracking) tracks the point at which the power is highest and uses those conditions for every panel in the array. Here the highest maximum power is for panel 1 which is 170.06 W. The corresponding voltage is 31.38 V. We would have to find out the current for every panel in the array when the voltage across the array is 31.38 V and consequently the power of every panel at that voltage. This can be done using the method previously used for a plain String Inverter.

The table below shows the calculated values

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Panel# | v | j | I | P |
| 1 | 0.70 | 0.94 | 5.42 | 170.06 |
| 2 | 0.74 | 0.91 | 4.82 | 151.13 |
| 3 | 0.71 | 0.94 | 5.24 | 164.55 |
| 4 | 0.72 | 0.92 | 5.27 | 165.26 |
| 5 | 0.70 | 0.94 | 5.18 | 162.63 |
| 6 | 0.72 | 0.93 | 5.28 | 165.79 |
| 7 | 0.73 | 0.92 | 5.04 | 158.06 |
| 8 | 0.73 | 0.92 | 5.18 | 162.67 |
| 9 | 0.75 | 0.90 | 4.67 | 146.66 |
| 10 | 0.74 | 0.91 | 5.18 | 162.53 |
| Effective Power |  |  | 1609.35 |

Thus the array has a total effective power of 1609.347 W when a String Inverter with MPPT is used.

iii) Micro-inverter: When a micro-inverter is used for every panel of an array, the panels seem to operate independently on their own and the array behaves like a parallel circuit and not a series circuit. The micro-inverter performs a MPPT operation on the panel it is connected to and provides the maximum output for each panel. Thus the total effective power of the array is nothing but the sum of maximum power of every panel.

The values used for making the I-V graph are not sufficient enough to determine a maximum power point for every panel. A precise calculation is needed. For that, we will calculate the maximum voltage Vmpp and maximum current Impp using the maximum normalized voltage vmpp and maximum normalized current density jmpp respectively.

From the fill factor module, we know that



jmpp = 

For m = 8.8 and n = 0.75 we get,

$$V\_{mpp}=\left(1+\frac{8.8}{0.75}\right)^{-\frac{1}{8.8}}=0.748$$

$$j\_{mpp}=\left(\frac{8.8}{0.75}\right)^{\frac{1}{0.75}}×\left(1+\frac{8.8}{0.75}\right)^{\frac{-1}{0.75}}=26.662×0.0336=0.896$$

Using $V\_{mpp}=v\_{mpp}×V\_{oc}$ and $I\_{mpp}=j\_{mpp}×I\_{sc}$we can get the values of maximum voltage and maximum current for every panel. $P\_{max}=V\_{mpp}×I\_{mpp}$ will give the maximum power for the corresponding panel. The data is tabulated below.

|  |  |  |  |
| --- | --- | --- | --- |
| Panel# | Vmpp | Impp | Pmax |
| 1 | 33.57 | 5.16 | 173.08 |
| 2 | 31.83 | 4.75 | 151.27 |
| 3 | 33.18 | 5.02 | 166.60 |
| 4 | 32.50 | 5.11 | 166.13 |
| 5 | 33.55 | 4.93 | 165.47 |
| 6 | 32.65 | 5.11 | 166.89 |
| 7 | 32.13 | 4.93 | 158.45 |
| 8 | 32.20 | 5.07 | 163.15 |
| 9 | 31.45 | 4.66 | 146.67 |
| 10 | 31.83 | 5.11 | 162.68 |
| Effective Power |  | 1620.38 |

Thus the total effective power of the array would be 1620.385 W if a micro-inverter is used for every panel.

By looking at the effective power of all the three inverters, we realize that an array having a micro-inverter attached to every panel would be beneficial since it provides a greater output. However the cost of such micro-inverters is greater than that of string inverters. Hence, string inverters are preferred if there is no obstruction to the irradiation available to the array. Using a String inverter with MPPT is often a good compromise. Attaching a micro-inverter to a set of two panels instead of attaching it to each and every panel is being experimented in some places to reduce the cost.

**Homework Problems**

1) In problem 2, determine and compare the precise effective power if a micro-inverter is used for a set of two panels instead of every panel. Panel 1 and Panel 2 will be attached together to a micro-inverter, Panel 3 and Panel 4 to another micro-inverter, and so on.

2) Estimate the effective power if the array is installed in such a way that the first 5 panels are connected in series to a String Inverter with MPPT and rest of the 5 panels are connected in series to just a String Inverter.